BULLETIN

OF THE

AMERICAN MATHEMATICAL SOCIETY

EDITED BY

EARLE R. HEDRICK

DAVID R. CURTISS WILLIAM R. LONGLEY

WITH THE ASSISTANCE OF

HAROLD T. DAVIS HARRY W. KUHN

THEOPHIL H. HILDEBRANDT DERRICK N. LEHMER

JOHN R. KLINE HENRY L. RIETZ

CAROLINE E. SEELY

VOLUME XL, NUMBER 3

MARCH, 1934

PUBLISHED BY THE SOCIETY

MENASHA, WIS., AND NEW YORK

1934

Whole No. 414 $9.00 a Year

PUBLISHED MONTHLY

Entered as second class matter April 24, 1926 at the post office at Menasha, Wis., under the act of August 24, 1912.

AMERICAN MATHEMATICAL SOCIETY

Mail Address: 501 West 116th Street, New York, N.Y., for the Secretary, Treasurer, Librarian, and Clerk.

Office: Room 413, Physics Building, Columbia University, New York City.

President: Professor A. B. Coble, University of Illinois, Urbana, Ill.

Secretary: Dean R. G. D. Richardson, Brown University, Providence, R.I.

Associate Secretaries: Professor M. H. Ingraham, University of Wisconsin, Madison, Wis.; Professor J. R. Kline, University of Pennsylvania, Philadelphia, Pa.; Dean T. M. Putnam, University of California, Berkeley, Calif.

Treasurer: Professor G. W. Mullins, Barnard College in Columbia University, New York City.

Librarian: Professor R. C. Archibald, Brown University, Providence, R.I.

Clerk: Dr. Caroline E. Seely, 413 Physics Building, Columbia University, New York City.

Editor-in-Chief of the Bulletin: Professor E. R. Hedrick, University of California at Los Angeles, Los Angeles, Calif.

Managing Editor of the Transactions: Professor R. D. Carmichael, University of Illinois, Urbana, Ill.

Chairman, Colloquium Editorial Committee: Professor Oswald Veblen, The Institute for Advanced Study, Princeton, N.J.

Managing Editor of the American Journal of Mathematics: Professor Abraham Cohen, Johns Hopkins University, Baltimore, Md.

Representatives of the Society in the Division of Physical Sciences of the National Research Council: Professors A. B. Coble, E. R. Hedrick, and Dean R. G. D. Richardson.

Representatives on the Council of the American Association for the Advancement of Science: Professors M. H. Ingraham and C. N. Moore.

Chairman, Committee on Arrangements for the Semi-Centennial: Professor T. S. Fiske, Columbia University, New York City.

THE ANNUAL MEETING IN CAMBRIDGE

The fortieth Annual Meeting of the American Mathematical Society was held at Harvard University and the Massachusetts Institute of Technology, Cambridge, Massachusetts, from Tuesday to Friday, December 26–29, 1933. The afternoon session on Thursday and the dinner were held at the Massachusetts Institute of Technology, the other sessions in the Alice Mary Longfellow Hall of Radcliffe College. In point of attendance of members and in number of papers offered, it was second only to the Annual Meeting of 1928 in New York City. The arrangements as made by the committee, of which Professor J. L. Walsh was chairman and Mrs. W. C. Graustein and Professors D. J. Struik and D. V. Widder the other local members, were carefully conceived and superbly executed, and the meeting proved to be one of the most interesting and pleasant in the annals of the Society. At the session on Friday morning on motion of Professor W. L. Hart the Society passed a hearty vote of thanks to the committee and the inviting institutions.

The Board of Trustees held a meeting in New York City on December 19; at the meeting called for 6:00 P.M. on December 26 there was no quorum present and adjournment was taken. The Council held a meeting at 8:00 P.M. on Wednesday and an adjourned session at the same hour on Friday.

The scientific meetings opened on Tuesday evening with a general session. On Wednesday morning one session was devoted to Analysis and one to a symposium of invited papers on the topic of Probability. At the latter the American Physical Society and Section B were guests; Professor D. J. Struik presided and four invited papers were presented as follows: *Remarks on causality and probability*, by Professor Eberhard Hopf, of the Massachusetts Institute of Technology; *Foundations of probability in the natural sciences*, by Professor Felix Bernstein, of Columbia University; *The probability of position in a canonical ensemble*, by Professor G. E. Uhlenbeck, of the University of Michigan; and *The Brownian motion*, by Professor Norbert Wiener, of the Massachusetts Institute of Technology. On account of illness, the fifth paper, on *Some analytical problems relating to probability*, which was to have been given by Dr.

Borge Jessen, of the Institute for Advanced Study at Princeton, was omitted. There was a lively discussion of these papers.

On Wednesday afternoon there were three sectional sessions. On Thursday morning there was scheduled the annual business meeting and election followed by a presentation of the Bôcher Memorial Prize. This prize was divided between Professors Marston Morse and Norbert Wiener and these gentlemen gave brief résumés of the papers on which the award was based. The memoir of the former, entitled *The foundations of a theory of the calculus of variations in the large in m-space*, was published in volume 32 of the Transactions of this Society, and that of the latter, entitled *Tauberian theorems*, was published in volume 33 of the Annals of Mathematics. Following this presentation and at the request of the Program Committee, Professor Georges Valiron, of the University of Paris, now exchange professor at Harvard University, delivered an address entitled *Schwarz's lemma; its extensions and applications.*

An important feature of the meetings was the symposium on General Analysis, held on Thursday afternoon, at which three papers were presented by invitation of the Program Committee: *On E. H. Moore's general analysis: the first theory*, by Professor T. H. Hildebrandt; *On E. H. Moore's general analysis: the second theory*, by Professor R. W. Barnard; and *A comparative survey of modern theories of functional analysis*, by Professor M. H. Stone.

On Friday morning, at the joint session with Section A of the A.A.A.S. and the Mathematical Association of America, Professor H. H. Mitchell, as retiring Vice-President of Section A, spoke on *Linear groups and finite geometries*, and Professor J. L. Coolidge, representing the Mathematical Association, spoke on *The rise and fall of projective geometry*. Friday afternoon and Saturday morning were devoted to the meetings of the Mathematical Association.

The annual dinner of the mathematical organizations, with more than three hundred in attendance, was held in the Walker Memorial Building of the Massachusetts Institute of Technology on Thursday evening, Professor J. L. Coolidge presiding and President K. T. Compton of the Institute giving a speech of welcome. In addition there were speeches by Presidents A. B. Coble of the Society and Arnold Dresden of the Association,

and Professors Archibald Henderson and Helen A. Merrill.

The Josiah Willard Gibbs Lecture was omitted this year.

At the invitation of Professor and Mrs. J. L. Coolidge the mathematicians were entertained at tea in Lowell House on Wednesday afternoon. There were many other entertainments also, including a visit to the Isabella Stewart Gardner Museum on Thursday and a tea given by courtesy of President Ada L. Comstock of Radcliffe College at Agassiz House. On Wednesday afternoon a complimentary concert was given by the Boston Symphony Orchestra in honor of the visiting members of the American Association for the Advancement of Science. After the session on Thursday afternoon there was an exhibition of the important machines, charts, and electrical integraph owned by the Massachusetts Institute of Technology.

The headquarters of the mathematicians was at the Hotel Commander, the Hotel Continental being used for overflow.

The attendance included the following two hundred fifty-four members:

C. R. Adams, R. B. Adams, R. P. Agnew, E. B. Allen, R. C. Archibald, C. S. Atchison, R. W. Babcock, F. H. Bailey, N. H. Ball, R. W. Barnard, Ralph Beatley, A. A. Bennett, Felix Bernstein, Garrett Birkhoff, G. D. Birkhoff, A. H. Black, L. M. Blumenthal, H. F. Bohnenblust, J. W. Bower, H. W. Brinkmann, Leonard Bristow, A. B. Brown, B. H. Brown, E. P. Brown, R. E. Bruce, S. S. Cairns, W. D. Cairns, R. H. Cameron, B. H. Camp, A. D. Campbell, G. A. Campbell, M. E. Carlen, W. B. Carver, W. F. Cheney, Alonzo Church, J. A. Clarkson, J. M. Clarkson, A. B. Coble, L. W. Cohen, Nancy Cole, E. G. H. Comfort, J. L. Coolidge, A. H. Copeland, L. P. Copeland, C. H. Currier, H. B. Curry, D. R. Curtiss, E. H. Cutler, H. M. Dadourian, D. R. Davis, J. W. Davis, C. E. Dimick, L. L. Dines, J. L. Doob, H. L. Dorwart, Jesse Douglas, Arnold Dresden, J. C. Durand, W. H. Durfee, L. A. Dye, Paul Eberhart, W. E. Ekman, W. W. Elliott, F. J. Feinler, M. M. Flood, Tomlinson Fort, M. C. Foster, C. W. Franklin, Philip Franklin, T. C. Fry, H. G. Funkhouser, A. S. Galbraith, C. A. Garabedian, J. J. Gergen, F. J. Gerst, B. C. Getchell, B. P. Gill, D. C. Gillespie, R. E. Gilman, M. C. Graustein, W. C. Graustein, B. F. Groat, H. V. Gummere, Margaret Gurney, J. G. Hardy, W. L. Hart, Alan Hazeltine, G. A. Hedlund, E. R. Hedrick, C. H. Helliwell, Archibald Henderson, M. R. Hestenes, H. C. Hicks, T. H. Hildebrandt, Einar Hille, Eberhard Hopf, Charles Hopkins, Ralph Hull, E. V. Huntington, W. A. Hurwitz, M. H. Ingraham, Dunham Jackson, Nathan Jacobson, R. L. Jeffery, R. A. Johnson, F. E. Johnston, B. W. Jones, E. R. van Kampen, A. E. Kennelly, L. S. Kennison, S. H. Kimball, J. R. Kline, Morris Kline, P.A. Knedler, B. O. Koopman, R. L. Korgen, H. L. Krall, H. K. Kutman, K. W. Lamson, A. W. Landers, M. K. Landers, A. E. Landry, C. G. Latimer, V. V. Latshaw, Solomon Lefschetz, D. H. Lehmer, A. M. M. Lehr, D. D. Leib, D. C. Lewis,

F. P. Lewis, Hans Lewy, R. B. Lindsay, G. H. Ling, L. L. Locke, W. R. Longley, E. R. Lorch, L. L. Lowenstein, J. J. Luck, N. H. McCoy, W. H. McEwen, James McGiffert, C. C. MacDuffee, Saunders MacLane, H. M. MacNeille, H. F. MacNeish, Wilhelm Maier, H. A. Merrill, Norman Miller, H. H. Mitchell, E. B. Mode, E. C. Molina, Deane Montgomery, C. N. Moore, R. K. Morley, Marston Morse, E. J. Moulton, S. B. Myers, M. M. Ness, E. P. Northrop, C. O. Oakley, E. G. Olds, Oystein Ore, F. W. Owens, F. W. Perkins, H. A. Perkins, H. B. Phillips, V. C. Poor, Hillel Poritsky, G. B. Price, S. M. Rambo, W. C. Randels, J. F. Randolph, W. R. Ransom, H. W. Raudenbush, C. J. Rees, M. S. Rees, R. G. D. Richardson, C. A. Richmond, D. E. Richmond, H. L. Rietz, David Rines, W. C. Risselman, H. P. Robertson, Robin Robinson, S. L. Robinson, M. F. Rosskopf, H. G. Russell, W. M. Rust, George Rutledge, M. F. Schmeiser, I. J. Schoenberg, H. E. Schoonmaker, C. H. W. Sedgewick, R. W. Sedgewick, Wladimir Seidel, Harlow Shapley, H. M. Sheffer, I. M. Sheffer, L. L. Silverman, James Singer, H. L. Slobin, A. H. Smith, C. E. Smith, I. W. Smith, T. L. Smith, Virgil Snyder, Joseph Spear, A. A. Stafford, M. E. Stark, M. H. Stone, R. E. Street, D. J. Struik, M. M. Sullivan, J. L. Synge, Otto Szasz, J. D. Tamarkin, J. H. Taylor, J. S. Taylor, J. M. Thomas, H. S. Thurston, Arthur Tilley, C. C. Torrance, M. M. Torrey, J. I. Tracey, W. J. Trjitzinsky, C. B. Tucker, A. W. Turner, F. E. Ulrich, J. L. Vanderslice, H. E. Wahlert, Warren Weaver, F. M. Weida, M. J. Weiss, M. E. Wells, V. H. Wells, Hermann Weyl, A. P. Wheeler, Hassler Whitney, J. K. Whittemore, D. V. Widder, Norbert Wiener, E. P. Wiggin, R. L. Wilder, S. S. Wilks, W. L. G. Williams, A. H. Wilson, E. B. Wilson, E. W. Wilson, W. A. Wilson, F. S. Woods, F. M. Wright, M. M. Young, S. D. Zeldin, Leo Zippin.

It was announced that the following persons had been elected to membership in the Society:

Mr. Vernon B. Bagnall, American Telephone and Telegraph Company, New York;
Professor Felix Bernstein, Columbia University;
Professor W. S. Claytor, West Virginia State College;
Major Chester Lawrence Fordney, United States Marine Corps, Chicago;
Dr. Hans Lewy, Brown University;
Mr. Everett Richard Matthews, University of California;
Professor Joseph P. Merrick, S. J., Baghdad College, Baghdad, Iraq;
Mr. Moses Richardson, Brooklyn College.

It was also announced that the following persons had been admitted to membership in the Society in accordance with reciprocity agreements:

Professor Leopold Fejér, University of Budapest;
Dr. Borge Jessen, Institute for Advanced Study, Princeton;
Professor Giovanni Ricci, Reale Scuola Normale Superiore, Pisa.

The ordinary membership in the Society is now 1783, including 50 nominees of sustaining members and 83 life members.

There are also 15 sustaining members. The total attendance of members at all meetings in 1933 was 1010; the number of papers read was 415; the number of members attending at least one meeting was 485.

At the annual election, which closed on December 28, and at which 208 votes were cast, the following officers were elected:

Vice-Presidents, Professors Marston Morse and H. S. Vandiver.

Secretary, Dean R. G. D. Richardson.

Treasurer, Professor G. W. Mullins.

Associate Secretaries, Professor M. H. Ingraham and Dean T. M. Putnam.

Member of the Editorial Committee of the Bulletin, Professor E. R. Hedrick.

Member of the Editorial Committee of the Transactions, Professor R. D. Carmichael.

Member of the Editorial Board of the American Journal of Mathematics, Professor E. T. Bell.

Member of the Editorial Committee of the Colloquium Publications, Professor E. T. Bell.

Members of the Council, Mr. J. R. Carson, Professors L. R. Ford, R. L. Jeffery, and Oystein Ore, and Dr. Warren Weaver, to serve three years; Professor L. E. Dickson, to serve one year.

The reports of the Treasurer and of the auditors (Professor H. W. Reddick and Mr. J. J. Tanzola) showed a balance of $2723.21, exclusive of the balances in the Bulletin, Transactions, Colloquium, Journal, Library, Sinking Fund, and special funds. The Society's Endowment Fund, invested in securities of par value $77,000, yielded in 1933 a net income of $3269.80; sustaining memberships for the year amounted to $1800. The amount received from sales of the Society's publications was $8781.58. During the year special contributions from members were received to the amount of $505.76. The trustees adopted a budget for 1934 showing estimated expenditures and receipts as $35,872.07 and $35,652.07, respectively. The Librarian reported that the Library of the Society now contains 8267 volumes, and that 50 volumes were received from the library of a New York engineer, Mr. E. U. Frey.

The following appointments were reported: as tellers for the election at the annual meeting, Professor C. O. Oakley and Dr.

H. W. Raudenbush; as representative on the National Research Council for the period beginning July 1, 1934, and to succeed Professor E. R. Hedrick, Professor Marston Morse; as representative on the Council of the American Association for the Advancement of Science to replace Professor Louis Ingold for 1933, Professor Tomlinson Fort; as representatives on the Council of the American Association for the Advancement of Science for 1934, Professors M. H. Ingraham and C. N. Moore; as the nominating committee, Professors G. D. Birkhoff (chairman), H. F. Blichfeldt, E. P. Lane, C. N. Moore, and J. H. M. Wedderburn; as committee on arrangements for the Pittsburgh meeting, Professors J. S. Taylor (chairman), C. S. Atchison, L. L. Dines, and J. R. Kline, Dr. W. I. Miller, and Professor E. G. Olds; as committee on the award of the Frank Nelson Cole Prize in Algebra in 1934, Professors L. E. Dickson (Chairman), E. T. Bell, and H. H. Mitchell.

An invitation from the University of Kentucky to hold a meeting of the Society at Lexington was received with thanks to the inviting institution.

Titles and cross references to the abstracts of papers read at the regular session follow below. Dr. Estes was introduced by Professor Franklin, Mr. Hammond by Professor Walsh, Mr. Steenrod by Professor Wilder, Professor Brauer by Professor Richardson, and Mr. Hummel by Professor MacDuffee. The joint paper by Professor Wall and Mr. Leighton was read by Mr. Leighton, that of Dr. Bochner and Professor Bohnenblust by Dr. Bochner, that of Professor Adams and Mr. Clarkson by Professor Adams, that of Dr. Doob and Professor Koopman by Dr. Doob, that of Professors Hille and Tamarkin by Professor Hille, and that of Professor Barnett and Dr. Nathan by Dr. Nathan.

The following is a list of presiding officers: Professor Marston Morse at the evening session on Tuesday; Professor E. R. Hedrick at the Wednesday morning session on Analysis, and Professor D. J. Struik at the session on Probability; on Wednesday afternoon at the session on Analysis, Professor Norbert Wiener, at that on Geometry, Topology, and Point Sets, Professors J. L. Coolidge and T. H. Hildebrandt, and at that on Algebra, Vice-President H. H. Mitchell; at the Thursday morning session, President A. B. Coble; at the afternoon session,

Professor Arnold Dresden; at the joint session on Friday, Vice-President C. N. Moore of Section A.

General Session, Tuesday Evening.

1. *On the dynamical theory of electrical commutator machines*, by Mr. W. H. Ingram. (Abstract No. 40-1-23-*t*.)

2. *The lift and the moment of an arbitrary aerofoil Joukovsky potential*, by Dr. G. G. Estes. (Abstract No. 40-1-24.)

3. *A new method for the evaluation of double integrals*, by Mr. W. L. Morris. (Abstract No. 40-1-25-*t*.)

4. *An extension of Cauchy's theorem*, by Dr. Wladimir Seidel. (Abstract No. 40-1-26.)

5. *Linear almost periodic transformations*, by Dr. R. H. Cameron (National Research Fellow). (Abstract No. 40-1-27.)

6. *On capacity in potential theory* (preliminary communication), by Mr. Jacques Hammond. (Abstract No. 40-1-28.)

7. *A numerical equivalent of the four-color problem*, by Dr. Hassler Whitney. (Abstract No. 40-1-29.)

8. *On double Riemann-Stieltjes integrals*, by Mr. J. A. Clarkson. (Abstract No. 39-9-247.)

9. *Derived numbers and approximate derivatives of non-measurable functions*, by Professor R. L. Jeffery. (Abstract No. 40-1-30-*t*.)

10. *The Dirichlet problem for domains with multiple boundary points*, by Dr. F. W. Perkins. (Abstract No. 40-1-31.)

11. *Some properties of arbitrary functions*, by Dr. Mabel Schmeiser. (Abstract No. 40-1-32.)

12. *The order of groups of automorphisms*, by Mr. Garrett Birkhoff (Junior Prize Fellow, Harvard University). (Abstract No. 39-11-305.)

13. *Abbreviated proofs in logic calculus*, by Mr. Saunders MacLane (Sterling Research Fellow). (Abstract No. 40-1-33.)

14. *The independence of estimates of variance in samples from Latin square lay-outs*, by Dr. S. S. Wilks. (Abstract No. 40-1-34-*t*.)

15. *Concerning certain reducible polynomials*, by Dr. H. L. Dorwart. (Abstract No. 40-1-35.)

16. *Note on a singular integral*, by Mr. E. P. Northrop. (Abstract No. 40-1-94.)

Section for Analysis, Wednesday Morning.

17. *On the transformation and convergence of continued fractions*, by Professor H. S. Wall and Mr. Walter Leighton. (Abstract No. 39-11-310.)

18. *On approximation to the solution of a normal system of ordinary linear differential equations*, by Professor W. C. Rissel-man. (Abstract No. 39-11-313.)

19. *Concerning some methods of "best approximation,"* by Professor I. M. Sheffer. (Abstract No. 40-1-36.)

20. *Polynomials of best approximation associated with certain problems in two dimensions*, by Professor W. H. McEwen. (Abstract No. 40-1-37.)

21. *On the periodic motions of dynamical systems with n degrees of freedom*, by Dr. D. C. Lewis, Jr. (National Research Fellow). (Abstract No. 40-1-38.)

22. *Semi-linear equations.* Part III: *Convex polygons*, by Professor C. O. Oakley. (Abstract No. 40-1-39.)

23. *Lagrange multipliers for functions of infinitely many variables*, by Professor L. W. Cohen. (Abstract No. 40-1-40.)

24. *On criteria for Fourier constants of L-integrable functions of two variables*, by Professor C. N. Moore. (Abstract No. 40-1-41.)

25. *Analytic functions with almost periodic coefficients*, by Dr. Salomon Bochner and Professor H. F. Bohnenblust. (Abstract No. 40-1-42.)

26. *Analytic theory of linear differential equations*, by Dr. W. J. Trjitzinsky. (Abstract No. 39-11-314.)

27. *Concerning measure and linear density properties of point sets* (preliminary communication), by Mr. J. F. Randolph. (Abstract No. 39-11-312.)

28. *Properties of functions $f(x, y)$ of bounded variation*, by Professor C. R. Adams and Mr. J. A. Clarkson. (Abstract No. 40-1-43.)

29. *The asymptotic solutions of certain linear ordinary differential equations of the second order*, by Professor R. E. Langer. (Abstract No. 40-1-93-*t*.)

30. *Note on the orthogonality of Tchebycheff polynomials on confocal ellipses*, by Professor J. L. Walsh. (Abstract No. 40-1-95-*t*.)

31. *On convergence in variation*, by Professor C. R. Adams and Mr. J. A. Clarkson. (Abstract No. 40-1-44-*t*.)

32. *Derivatives of higher order as single limits*, by Professor Philip Franklin. (Abstract No. 40-1-45-*t*.)

33. *On H-functions and the distribution functions associated with them*, by Dr. E. K. Haviland. (Abstract No. 40-1-46-*t*.)

34. *Polygenic functional solutions of certain types of integral equations*, by Professor V. C. Poor. (Abstract No. 40-1-47-*t*.)

35. *On the tetradite Ψ-function with application to apportionment theory*, by Dr. W. R. Thompson. (Abstract No. 40-1-48-*t*.)

36. *Recursion formulas for the tetradite Ψ and the incomplete I-function*, by Dr. W. R. Thompson. (Abstract No. 40-1-49-*t*.)

37. *The general case of linear integro-differential equations*, by Dr. W. J. Trjitzinsky. (Abstract No. 39-11-315-*t*.)

38. *Functions differentiable on the boundaries of regions*, by Dr. Hassler Whitney. (Abstract No. 39-11-316-*t*.)

39. *Derivatives, difference quotients, and Taylor's formula*. II, by Dr. Hassler Whitney. (Abstract No. 39-11-317-*t*.)

40. *An auxiliary theorem associated with the calculus of variations*, by Dr. W. T. Reid. (Abstract No. 40-1-50-*t*.)

Section I, Analysis, Wednesday Afternoon.

41. *Integral equations and elliptic functions*, by Professor Wilhelm Maier. (Abstract No. 40-1-51.)

42. *Certain fundamental notions and theorems in polygenic function theory*, by Professor V. C. Poor. (Abstract No. 40-1-52.)

43. *Note on relations connecting certain cases of convergence in the mean*, by Professor Dunham Jackson. (Abstract No. 40-1-53.)

44. *Instability and transitivity*, by Professor Marston Morse. (Abstract No. 40-1-54.)

45. *A necessary and sufficient condition for a regular point*, by Professor G. C. Evans. (Abstract No. 40-1-55-*t*.)

46. *The resolvent of a self-adjoint transformation*, by Dr. J. L. Doob (National Research Fellow) and Professor B. O. Koopman. (Abstract No. 39-11-307.)

47. *On the theory of Laplace integrals*. II, by Professors Einar Hille and J. D. Tamarkin. (Abstract No. 40-1-56.)

48. *Necessary and sufficient conditions for the representation of a function by a doubly infinite Laplace integral*, by Professor D. V. Widder. (Abstract No. 40-1-57.)

49. *On the summability of the conjugate series*, by Mr. W. C. Randels. (Abstract No. 40-1-58.)

50. *Some inequalities for non-uniformly bounded ortho-normal polynomials* (preliminary communication), by Mr. M. F. Rosskopf. (Abstract No. 40-1-59.)

51. *On some asymptotic relations for the characteristic values of elliptic differential equations*, by Dr. H. L. Krall. (Abstract No. 40-1-92.)

52. *Linear transformations in function space*, by Professor I. A. Barnett and Dr. D. S. Nathan (National Research Fellow). (Abstract No. 40-1-96.)

Section II, Geometry, Topology, and Point Sets, Wednesday Afternoon.

53. *On the ordered quadrilaterals in- and circumscribed to the plane rational quartic curve* (R_2^4) *with triangular symmetry* (preliminary communication), by Professor A. E. Landry. (Abstract No. 40-1-60.)

54. *Equiareal maps with conic meridians and parallels*, by Professor B. H. Brown. (Abstract No. 39-11-296.)

55. *Transitive geodesics on closed orientable surfaces of genus greater than one*, by Dr. G. A. Hedlund (National Research Fellow). (Abstract No. 40-1-61.)

56. *On n-dimensional differential geometry in the small and in the large*, by Dr. S. B. Myers. (Abstract No. 39-11-311.)

57. *Geodesics on polyhedral surfaces*, by Professor Philip Franklin. (Abstract No. 40-1-62.)

58. *On the deviation of geodesics and null-geodesics particularly in relation to the properties of spaces of constant curvature and indefinite line-element*, by Professor J. L. Synge. (Abstract No. 40-1-63.)

59. *A characterization and generalization, by internal properties alone, of those open subsets of* E_2 *whose boundaries are manifolds*, by Professor R. L. Wilder. (Abstract No. 39-11-319-*t*.)

60. *Cyclic elements of higher orders*, by Dr. G. T. Whyburn. (Abstract No. 39-11-318-*t*.)

61. *The intersection of chains on a topological manifold*, by Dr. W. W. Flexner. (Abstract No. 40-1-64-*t*.)

62. *Operations on plane sets*, by Dr. Deane Montgomery (National Research Fellow). (Abstract No. 40-1-65.)

63. *Characterizations of certain finite curve-sums*, by Mr. N. E. Steenrod. (Abstract No. 40-1-66.)

64. *Completely regular approximations to regular manifolds*, by Professor S. S. Cairns. (Abstract No. 40-1-67.)

65. *Further non-involutorial Cremona space transformations contained in a special linear complex*, by Dr. A. H. Black. (Abstract No. 40-1-68.)

66. *Semi-Peanian spaces*, by Dr. Leo Zippin. (Abstract No. 40-1-97-*t*.)

67. *Topological characterizations of 2-dimensional manifolds*, by Dr. E. R. van Kampen. (Abstract No. 40-1-98.)

68. *A theorem on fixed points*, by Professor P. A. Smith. (Abstract No. 40-1-99-*t*.)

69. *The topology of transformation-sets*, by Mr. Garrett Birkhoff (Junior Prize Fellow, Harvard University). (Abstract No. 40-1-69-*t*.)

70. *Hausdorff groupoids*, by Mr. Garrett Birkhoff (Junior Prize Fellow, Harvard University). (Abstract No. 40-1-70 *t*.)

71. *Linearly connected manifolds and ennuples of curves*, by Professor Harry Levy. (Abstract No. 40-1-71-*t*.)

72. *Curvatures in Riemannian space*, by Professor Harry Levy. (Abstract No. 40-1-72-*t*.)

73. *The Bertini transformations of space*, by Professor F. R. Sharpe and Dr. L. A. Dye. (Abstract No. 40-1-73-*t*.)

74. *Mechanical models of spaces with positive-definite line-element*, by Professor J. L. Synge. (Abstract No. 40-1-74-*t*.)

Section III, Algebra, Foundations, and Statistics, Wednesday Afternoon.

75. *A determination of all cyclotomic quintic fields*, by Dr. Ralph Hull (National Research Fellow). (Abstract No. 39-11-308.)

76. *A test for equivalence of positive quaternary quadratic forms*, by Professor B. W. Jones. (Abstract No. 40-1-75.)

77. *Independent postulates for an "informal Principia system with equality,"* by Professor E. V. Huntington. (Abstract No. 40-1-76.)

78. *Klein's theory of algebraic equations and its connection with the theory of algebras*, by Professor Richard Brauer. (Abstract No. 40-1-77.)

79. *Note on the class number in a rational semi-simple algebra*, by Professor C. G. Latimer. (Abstract No. 40-1-78.)

80. *Ideal theory and partial differential equations*, by Dr. H. W. Raudenbush. (Abstract No. 40-1-79.)

81. *Lacunary recurrences for Bernoulli numbers*, by Dr. D. H. Lehmer. (Abstract No. 40-1-80.)

82. *On simply transitive primitive groups*, by Professor Marie J. Weiss. (Abstract No. 40-1-81.)

83. *On continued fractions of order n*, by Mr. P. M. Hummel. (Abstract No. 39-11-309.)

84. *Division by non-singular matric polynomials*, by Mr. M. M. Flood. (Abstract No. 40-1-82.)

85. *On the theory of invariants of n-planes*, by Professor Lennie P. Copeland. (Abstract No. 39-11-306.)

86. *A determinant theorem obtained from the characterization of pseudo r-spheric sets*, by Dr. L. M. Blumenthal (National Research Fellow). (Abstract No. 40-1-83.)

87. *Metabelian groups of order p^m*, by Dr. Charles Hopkins. (Abstract No. 40-1-100.)

88. *Correlation of indices*, by Dr. A. T. Craig. (Abstract No. 40-1-84-*t*.)

89. *Note on the moments of a Bernoulli distribution*, by Dr. A. T. Craig. (Abstract No. 40-1-85-*t*.)

90. *Problems of structure-theoretic type in mathematical logic*, by Mr. Saunders MacLane (Sterling Research Fellow). (Abstract No. 40-1-86-*t*.)

91. *General properties of algebraic systems*, by Mr. Saunders MacLane (Sterling Research Fellow). (Abstract No. 40-1-87-*t*.)

92. *On a certain algebra of quantum mechanics*, by Professor A. A. Albert. (Abstract No. 40-1-88-*t*.)

93. *On cyclic equations of prime degree*, by Professor A. A. Albert. (Abstract No. 40-1-89-*t*.)

94. *The value of the constants in Waring's problem*, by Dr. R. D. James (National Research Fellow). (Abstract No. 40-1-90-*t*.)

95. *An interpolation formula which when adjusted for rational centering of data proves to be osculatory*, by Professor C. H. Forsyth. (Abstract No. 40-1-91-*t*.)

R. G. D. RICHARDSON,
Secretary

INTERNATIONAL MATHEMATICAL CONGRESS MEDALS

In September, 1932, the International Congress of Mathematicians at Zurich approved the foundation of two gold medals, to be awarded at successive Congresses for outstanding achievements in mathematics. The suggestion was due to the late Dr. J. C. Fields, Research Professor of Mathematics at the University of Toronto, who wished to dispose in this way of a balance of funds remaining after the publication of the Proceedings of the Toronto Congress, 1924. The Zurich Congress appointed an

international Committee, consisting of Professors Severi (President), Birkhoff, Carathéodory, Cartan, Takagi, to decide on the awards to be made at the Congress at Oslo in 1936.

The task of designing a suitable medal was entrusted to the distinguished Canadian sculptor, Dr. R. Tait McKenzie, R.C.A., who has now completed his work.

The medal is two and a half inches in diameter. The obverse shows the head of Archimedes facing right. As there are no authentic portraits of this perhaps greatest of all mathematicians, recourse was had to the fine collection of over thirty pictures collected by Professor David Eugene Smith, and placed by him in Columbia University. They show the ideas of as many artists, ancient and modern, of what his appearance may have been. They naturally vary greatly, so the sculptor followed his own impression from reading his life and works. He shows the

sage as a man of mature age, vigorous, with curly hair and beard, straight Greek nose and prominent brow. In the field is the word "Archimedous," (portrait) of Archimedes, in Greek capitals, and the artist's monogram, "RTM," and "MCMXXXIII."

The inscription surrounding it is, "Transire suum pectus mundoque potiri," which may be freely translated: "To transcend one's human limitations and master the universe." This appropriate quotation from the Roman poet Manilius was supplied by Professor Norwood of the University of Toronto.

The reverse has a label bearing the inscription, "Congregati ex toto orbe mathematici ob scripta insignia tribuere" which may be freely translated: "Mathematicians gathered together from the whole world honour noteworthy contributions to knowledge."

Behind the label is a laurel-branch, and cut in the background can be made out the diagram of a sphere contained in a cylinder. The determination of the relation of these two was one of the outstanding achievements of Archimedes, and the diagram was engraved on his tomb. The name of the recipient will be cut on the edge of the medal and will not interfere with the design.

J. L. SYNGE

THE UNIVERSITY OF TORONTO

VEBLEN ON PROJECTIVE RELATIVITY

Projektive Relativitätstheorie. By O. Veblen. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 2, Heft 1. Berlin, Springer, 1933. 6+73 pp.

This book by Professor Veblen is a result of a series of lectures given at the University of Göttingen during the summer of 1932. It deals with that new aspect of the theory of relativity which is often called projective relativity on account of its relation to projective geometry. It allows a unified theory of the gravitational and the electromagnetic field, and, though it is not referred to in Veblen's book, also offers a possibility of including modern wave mechanics. Apart from this it throws a new light on such an old theory as classical projective geometry. The material is mainly taken from papers by the author himself and by close collaborators. The exposition is elegant and clear.

The present theory is therefore a result of two series of investigations, one mathematical and one physical. The mathematical side is the theory of projective connections, the physical side consists in the many attempts, begun by H. Weyl in 1918, to define a space-time structure depending not only on gravitational but also on electromagnetic potentials. Projective relativity seems to offer a rather simple and attractive solution.

The theory of projective connections is a generalization of projective geometry in the same sense as Riemannian geometry is a generalization of euclidean geometry. It is a theory of manifolds for which ordinary projective relations exist in the immediate neighborhood of a generating point, these relations being connected by a law which makes the manifold a "curved" projective manifold.

This is done in the following way. In a four-dimensional manifold with coordinates x^i, $i=1, 2, 3, 4$, there belongs to every point a "tangential space" of the dx^i which can be considered as an affine space. In a Riemannian geometry of fundamental tensor g_{ij} we have, at every point, a "light cone" $g_{ij}dx^i dx^j=0$ in the tangential space. We now introduce a non-degenerate quadric of which this cone is the asymptotic cone, and it is possible to define in each tangential space a non-euclidean geometry with respect to this quadric. The projective differential geometry of this kind of relativity is the "curved" generalization of this geometry.

To master the properties homogeneous coordinates are introduced into the space of the dx^i through the relations

$$dx^i = X^i/(\phi_\alpha X^\alpha), \quad (i = 1, 2, 3, 4;\ \alpha = 0, 1, 2, 3, 4);$$

the ϕ_α are functions which allow us to write for the hyperplane at infinity the equation $\phi_\alpha X^\alpha=0$. Homogeneous coordinates do not change when they are multiplied by a factor e^{x^0} of the coordinates (x^1, x^2, x^3, x^4). With respect to the transformations

$$\bar{x}^0 = x^0 + \log \rho, \quad \bar{x}^i = \bar{x}^i(x), \ (\rho \text{ a function of } x^1, \cdots, x^4)$$

we can then define "projective tensors," for example,

$$\bar{G}_{\alpha\beta} = G_{\sigma\tau}\frac{\partial x^\sigma}{\partial \bar{x}^\alpha}\frac{\partial x^\tau}{\partial \bar{x}^\beta}, \qquad (\alpha, \beta = 0, 1, 2, 3, 4).$$

From the equations $X^i = kdx^i$, $X^0 = k(1 - \phi_i dx^i)$, where k is an arbitrary number, the transformations of the X^α can be found:

$$\bar{X}^i = Kd\bar{x}^i = \frac{K}{k}X^j\frac{\partial \bar{x}^i}{\partial x^j}, \quad \bar{X}^0 = K(1 - \bar{\phi}_i d\bar{x}^i) = K(1 - \phi_i dx^i) = \frac{K}{k}X^0,$$

where K is a proportionality factor.

In the tangential space the equation $G_{\alpha\beta}X^\alpha X^\beta = 0$ determines a quadric, which can be used for the determination of a non-euclidean metric. The tensor $G_{\alpha\beta}$ "contains" a projective scalar ϕ, a projective vector ϕ_α and an affine tensor g_{ij}, according to the formula

$$G_{\alpha\beta} = \phi^2(g_{\alpha\beta} + \phi_\alpha\phi_\beta) = \phi^2\gamma_{\alpha\beta},$$
$$G_{00} = \phi^2, \quad G_{i\beta}/G_{00} = \gamma_{\alpha\beta}, \quad G_{0\alpha}/G_{00} = \phi_\alpha.$$

The g_{ij} and ϕ_α can be used to determine gravitational and electromagnetic potentials.

The next task is to relate the local spaces by means of a connection. First it is shown how classical projective geometry can be obtained. Here exist preferred coordinate systems Z^α, "projective coordinate systems," connected by transformation formulas of the kind $\bar{Z}^\alpha = p^\alpha Z^\beta$ which are related to the x^i by means of the equations $Z^\alpha = e^{x^0}f^\alpha(x^1, x^2, x^3, x^4)$. Classical projective geometry is now characterized by the existence of a family of projective scalars $Z = p_\alpha A^\alpha$ with arbitrary constants p_α. Elimination of the p_α leads to the differential equations

$$\frac{\partial^2 Z}{\partial x^\alpha \partial x^\beta} - \Pi^\sigma_{\alpha\beta}\frac{\partial Z}{\partial x^\sigma} = 0, \quad \frac{\partial Z}{\partial x^0} = Z,$$

for which the author suggests the name of "differential equations of projective geometry." The functions $\Pi^\sigma_{\alpha\beta}$ transform under a transformation of the x^0, x^i as the parameters of connection of an affine five-dimensional manifold:

$$\bar{\Pi}^\alpha_{\beta\gamma} = \left(\Pi^\sigma_{\rho\tau}\frac{\partial x^\rho}{\partial \bar{x}^\beta}\frac{\partial x^\tau}{\partial \bar{x}^\gamma} + \frac{\partial^2 x^\sigma}{\partial \bar{x}^\beta \partial \bar{x}^\gamma}\right)\frac{\partial \bar{x}^\alpha}{\partial x^\sigma}.$$

The $\Pi^\alpha_{\beta\gamma}$ define a special type of projective connection.

The differential equations for Z are not the most general of their kind, because they satisfy special integrability conditions. Their curvature tensor $R^\lambda_{\alpha\beta\gamma}$ must vanish. If we discard these integrability conditions, we obtain the general "curved" projective geometry, also with parameters of connection $\Pi^\alpha_{\beta\gamma}$. The author defines this in detail, and also shows how the transition to non-homogeneous coordinates can be made. He then shows how infinitesimal projective displacements can be determined, where the $\Lambda^\sigma_{\alpha\beta}$ depend on $\Pi^\sigma_{\alpha\beta}$ (but not uniquely):

$$\frac{\partial X^\alpha}{\partial x^\sigma} + \Lambda^\alpha_{\beta\sigma}X^\beta = 0.$$

It is important to observe that in projective differential geometry connection ("Zusammenhang") and displacement ("Uebertragung") must be separately treated. The connection determines only under certain circumstances a displacement. There is also a difficulty with the generalization of the notion of path, as the coordinates X^α determine a point, not a vector. Paths are here defined as curves for which points in the tangential space pass by displacement into themselves.

The projective connection and its displacements can now be made to depend on $G_{\alpha\beta}$. A special case is the non-euclidean geometry of the Cayley type, which finds extensive description. Generalization to the theory of a general projective tensor of second order leads to a "generalized theory of conic sections," which not only contains a Riemannian geometry, but also contains non-metrical properties. This means in the physical interpretation that gravitational and electromagnetic elements are possible.

The geometry is now ready for physical interpretation. The "paths" of the projective displacement defined by $G_{\alpha\beta}$ become the world lines of an electrical particle; they also include the geodesic lines of the metric. A variation principle

$$\delta \int B g^{1/2} dx^1 dx^2 dx^3 dx^4 = 0,$$

where B is the curvature scalar belonging to $\gamma_{\alpha\beta}$ and the variation is carried out with respect to $\gamma_{\alpha\beta}$ with $\gamma_{00} = 1$, gives the field equations for empty space. When they are split into their affine parts they yield the ordinary field equations of relativity. The physical theory only uses the g_{ij}, ϕ_i parts of $G_{\alpha\beta}$, not the ϕ. The author remarks that it may be possible to connect this ϕ with the Schrödinger wave theory.

The last chapter brings a five-dimensional interpretation. A bibliography is found at the end.

The author has given an exposition of the relations between his theory and wave mechanics in more recent papers on the *Geometry of two-component spinors* and the *Geometry of four-component spinors* (Proceedings of the National Academy of Sciences, April 19, 1933, pp. 462–474, 503–517). These papers, together with the book on projective relativity, present a complete unified field theory.

Veblen's interpretation of projective relativity has many points in common with and also certain differences from the similar theory presented by Schouten and Van Dantzig in several recent papers published in the Zeitschrift für Physik (see also Annals of Mathematics, vol. 34 (1933), p. 271). One of the chief points of difference between the two theories, which both interpret gravitation, electromagnetism, and wave mechanics, lies in the fact that the latter authors use a projective connection with a set of five equivalent homogeneous coordinates, while in Veblen's interpretation one coordinate, x^0, is singled out from the beginning. Both theories can be made to embrace previous attempts to construct a "curved" projective geometry and a projective relativity.

D. J. Struik

RADÓ ON PLATEAU'S PROBLEM

On the Problem of Plateau. By Tibor Radó. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 2, Heft 2. Berlin, Springer, 1933. 109 pp.

This report is an interesting and informative account of the recent work on the problem of Plateau—the problem which requires a minimal surface, or surface of least area, bounded by a given closed curve—dealing especially with the contributions made in recent years by Garnier, Douglas, Radó, and McShane. The survey consists of six chapters.

Chapter 1 reviews some general notions concerning length and area, with the implications of the various definitions that have been proposed for the latter. The definition of area most significant for the work on the Plateau problem is that given by Lebesgue in his thesis: the area of a surface S is the *least* limit towards which the area of a polyhedral surface P can tend when P approaches to S. This definition is associated with the essential lower semi-continuity property of area.

The importance for the problem of Plateau of monotonic transformations of the unit circumference C into the given contour Γ is pointed out, that is, the totality of ways of representing Γ as a topological image of C. The introduction of these monotonic transformations into the problem, as well as the important remark of the *compactness* of the totality of them, is due to the present reviewer in his abstracts in this Bulletin and his European lectures of 1929. This property of compactness makes it possible to prove the attainment of the minimum of any lower semi-continuous functional having these monotonic transformations as range of the argument, by a simple and classical procedure going back to Weierstrass and Fréchet. The totality of surfaces S bounded by a given contour does not form a compact set; it was the necessity of obtaining such a set as range of the argument of the functional, area of S, that underlay the restrictions to which the contour was subjected in the earlier work, notably that of Lebesgue and Haar.

In Chapter 2 are reviewed some facts from the differential geometry of minimal surfaces, necessary in the sequel, especially the formulas of Monge, Weierstrass, and Schwarz.

Chapter 3 begins with five modes of formulation $P_1, \cdots, P_5$ of the Plateau problem, varying somewhat in their implications. In these formulations a minimal surface is taken to be one defined by the Weierstrass formulas

$$x_i = \Re\phi_i(w), \quad \sum_{i=1}^{3} \phi_i'^2(w) = 0,$$

which amount to the condition of zero mean curvature for the surface together with the requirement of its conformal mapping on a circular disc.

If we require a surface which not only is minimal in the sense of the preceding formulas, but also has least area, then we have what Radó calls the "simultaneous problem." Examples, due to Schwarz, are cited of minimal surfaces which do not have the least area property in their entire extent.

The importance of singular or branch points for the formulation and solution of the problem is emphasized. A branch point is one where $\phi_i'(w)=0$, for $i=1, 2, 3$, and is said to be of nth order if the derivatives of each ϕ_i up to the nth order inclusive vanish, while the same is not true for the $(n+1)$th order.

An interesting theorem, due to Radó, is that if the contour Γ has a simply-covered *star-shaped* central or parallel projection on some plane, then a minimal surface bounded by Γ cannot have branch points. A plane curve is called star-shaped if there exists a point O in its plane such that every ray issuing from O intersects the curve in one and only one point. All convex curves are star-shaped, but not conversely.

The various modes of formulation of the Plateau problem lead to various uniqueness questions, in answer to which only some partial results have been obtained. Radó has proved the theorem: if Γ has a simply-covered *convex* central projection on some plane, then there is one and only one minimal surface bounded by Γ. A consequence is that the minimal surface bounded by any given skew quadrilateral is unique, a theorem going back to Schwarz.

That a given contour may bound several distinct minimal surfaces, of the topological type of a circular disc, is shown by examples due to Schwarz, Radó, and Wiener. In the series of statements $P_1, \cdots, P_5$ of the Plateau problem, the first four assume the surface in parametric form, while P_5 requires the surface in the form $z=f(x, y)$ and gives the contour with a simply-covered orthogonal projection on the xy-plane. Chapter 4 deals with this non-parametric form of the problem, where, besides, Γ is subject to a certain "three-point condition," whose chief implication is that the orthogonal projection of Γ on the xy-plane is convex. Chapter 5 begins the treatment of the recent work, where the contour has a general shape, and the surface is assumed in parametric form. The solutions of Garnier, Douglas, and Radó are discussed.

Garnier's result is subject to the restriction that the contour *shall have no knots*, and shall consist of a finite number of arcs each with bounded curvature. Douglas' result, the most general, allows Γ to be any Jordan curve whatever, and indeed, in euclidean space of an arbitrary number n of dimensions, the value of n having no influence on either method or result.

Radó's result restricts the contour to be capable of bounding at least one surface of finite area. This condition will not be satisfied if the contour contains a spiral which contracts with a sufficient degree of slowness, for instance, the analytic curve (given by the reviewer†) which is defined in spherical polar coordinates by the equations $r=\cos\phi$, $\theta=\tan^{\frac{1}{2}}\phi$, ($\phi$ latitude, θ longitude).

As to method, Garnier follows the classic program of Riemann and Weierstrass, involving solution of the so-called monodromy group problem in the theory of linear differential equations of the second order.

Douglas' method is based on the functional $A(g)$ introduced by him, whose argument g is an arbitrary representation $x_i=g_i(\theta)$ of Γ as topological image of the unit circumference C, and whose definition is

$$A(g)=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{2\pi}\left(\frac{l^*}{l}\right)^2 d\theta d\phi,$$

† J. Douglas, Proceedings of the National Academy of Sciences, vol. 19 (1933), pp. 448–451.

where l^*, l are the lengths of corresponding chords of Γ and C. The function $A(g)$ is amenable to immediate treatment by the same method which Weierstrass gave to prove the attainment of the minimum of a continuous function of a real variable. As brought out by Fréchet in his thesis, this reasoning requires only the two postulates that the function be lower semi-continuous and its argument range compact, and both these postulates are satisfied by $A(g)$.

Radó's method is based on constructing a sequence of polyhedral surfaces whose boundaries tend to Γ and whose areas tend to $m(\Gamma)$, the minimum of simply-connected areas bounded by Γ, and then representing these polyhedra conformally on the unit circular disc. Thus Radó's solution presumes the theory of conformal mapping.

On the other hand, it is an essential feature of the work of Douglas that when $n=2$ and Γ becomes a plane curve, his method gives a conformal map of the interior of Γ on the unit circular disc, attaching continuously to a topological correspondence between Γ and the unit circumference, that is, an independent proof of the classic Riemann mapping theorem together with the boundary correspondence theorem of Osgood and Carathéodory.

Chapter 6 deals first with the simultaneous problem concerned with the least area property of the minimal surface whose existence is established by the preceding methods. The contributions of Radó and Douglas are described, followed by a discussion of the new work of McShane, one of whose essential features is to operate only in the class of saddle-surfaces and to utilize certain selection theorems applying to such surfaces.

The report concludes with an account of the work of Douglas on the minimal surface bounded by two given non-intersecting contours, and on one-sided minimal surfaces with a given bounding curve.

In the latter theory, one of the theorems proved by the present reviewer is that if the simply-connected minimal surface determined by Γ has in its interior a branch point like that of the Riemann surface for the $2n$th root of z, then a one-sided minimal surface bounded by Γ exists. The published paper† containing this theorem includes a diagram by way of illustration. Radó attributes to the reviewer the inference that every contour whose *orthogonal projection* has the general shape of this illustration will give a simply-connected minimal surface with a branch point like that of $z^{1/2}$. This inference, together with the difficulties to which it leads, is nowhere stated nor implied in the reviewer's text.

Some corrections are as follows. Page 104, third line from bottom, instead of $4\pi\alpha^2$ read $\pi\alpha^2$. The same correction in formula (6.17) on page 105. In formulas (6.18), (6.19), strike out the term $3\pi\alpha^2$. Page 79, last line, write $i\lambda$ in place of λ. Corresponding to the two transformations at the bottom of page 79, $A(T)$ gives two functions $J_1(\lambda)$, $J_2(\lambda)$. In the second displayed formula on page 80, replace $J'(0)$ by $\frac{1}{2}[J_1'(0)-iJ_2'(0)]$.

The tract is clearly written, contains much information presented in a compact style, and should stimulate interest in its important field, where much valuable work still remains to be done.

JESSE DOUGLAS

† J. Douglas, *One-sided minimal surfaces with a given boundary*, Transactions of this Society, vol. 34 (1932), pp. 731–756; diagram on p. 733.

THE CALIFORNIA COLLOQUIUM

Differential Equations from the Algebraic Standpoint. By J. F. Ritt. New York, American Mathematical Society, 1932. x+172 pp.

The author sets himself the task of developing a theory of elimination which will reduce the existence problem for a finite or infinite system of algebraic differential equations to the application of the implicit function theorem taken with Cauchy's theorem in the ordinary case and Riquier's in the partial. The number of unknowns and independent variables is considered finite and fixed at the outset. The left members of the system are polynomials in the unknowns and their derivatives. The number of derivatives entering a given polynomial, presumably, is finite, though the number varies from polynomial to polynomial and may increase without limit in a sequence of polynomials.

The quest being, essentially, for holomorphic solutions of systems with holomorphic coefficients, every solution must satisfy what Riquier has named the *prolonged system*, which comprises all equations arising by differentiating the given equations. The author calls an algebraic differential system reducible or irreducible according as its prolonged system is algebraically reducible or not. One of the main currents of thought in the book seems to spring from the remark that an algebraically irreducible system may be reducible when regarded as a differential system.

The course followed by the author in accomplishing his purpose will now be indicated in summary fashion. It is shown that every system Σ is equivalent to a finite number of closed irreducible systems

$$\Sigma_1, \Sigma_2, \cdots, \Sigma_s \tag{1}$$

in the sense that every solution of Σ satisfies at least one Σ_i and every solution of a Σ_i satisfies Σ. For the case of a finite system, a theoretical process is developed (§67) for obtaining each of these irreducible systems in terms of a corresponding *basic set* B_i, which contains at most one equation corresponding to each unknown and in addition has other special defining properties. A *normal solution* of B_i is one for which none of a certain associated set of polynomials, called the *separants*, vanishes identically. Every solution of Σ_i satisfies the corresponding B_i, and, in the nature of a partial converse, it is proved that every normal solution of B_i satisfies the corresponding Σ_i (§66). Furthermore, every solution of Σ_i which is not normal for B_i satisfies a system Σ_{ij} which has a basic set B_{ij} of lower rank than B_i. This makes it possible to replace (1) by a like finite set having the additional property that every solution of the original system Σ is a normal solution for at least one of the basic sets which have been associated with the Σ_i.

The above, which applies literally only to the ordinary case, also describes the author's treatment of the partial case, if normal solution is replaced by regular, the latter type being one for which the set of non-vanishing polynomials has been augmented by the corresponding *initials*.

The final step is the development of a theoretical process for finding a finite set of equations equivalent to a closed irreducible system Σ_i. This is accom-

plished by the extension of the Hilbert-Netto theorem to differential systems. As the author remarks, it remains to supplement the process by giving a means of specifying a priori an upper limit to the number of steps necessary in its application to a given system. The author determines a limit in the particular case of the closed irreducible system Σ_1 which gives the general solution for an algebraically irreducible equation of the first order in a single unknown and a single independent variable.

We turn now to a discussion of some details and by-products of the treatment. The elimination process employed is Euclid's division algorithm, which runs through the whole book, giving to the method a unity at times bordering on monotony. The reader is apt to wonder why the resolvent system for polynomials is not systematically employed. Its use in certain cases would be more efficient. For example, if one quadratic polynomial with literal coefficients is divided by another, the remainder is the resultant multiplied by an extraneous factor, which in the present terminology is the initial of the divisor (see Fricke, *Algebra*, vol. 1, p. 16). At the same time, this perhaps gives the reason why the division algorithm is not used in algebra to obtain the resultant.

As the author is of course aware (see his remarks on p. 62), there is a mixture of algebraic and function-theoretic methods in the book. Consequently the algebraic nature of a result is sometimes obscured by the analytic means of obtaining it. Related to this is the fact that the coefficients are taken from a certain field throughout. It is expedient to make thus assumption when an existence theorem is to be applied, but in the application of the division algorithm, for example, it is merely necessary that the coefficients belong to a ring which is closed under differentiation.

The author divides the unknowns into two sets, one of which he names "arbitrary." Names for both sets in keeping with the literature are *principal* and *parametric*. The two sets are characterized as follows: the system implies no equation free from principal unknowns, and implies, corresponding to each principal unknown, at least one equation containing it and no other principal unknown. In the ordinary case, let a set of principal unknowns be $y_1, \cdots, y_p$. There is a basic set $A_1, \cdots, A_p$ for the corresponding closed system such that each A involves the y with the same index and no y with index higher than its own. An equation

$$w - \lambda_1 y_1 - \cdots - \lambda_p y_p = 0, \tag{2}$$

linear in the principal unknowns and an auxiliary unknown w, is adjoined to the system. The augmented system has a basic set $R, B_1, \cdots, B_p$ having the same properties with respect to $w, y_1, \cdots, y_p$ as the A's with respect to $y_1, \cdots, y_p$. The circumstance that (2) is linear and of order zero has for effect that each B is of order zero and linear in the y of equal index. Consequently all the principal unknowns are expressible rationally in terms of the parametric unknowns, the auxiliary unknown, and their derivatives. The equation $R=0$ determines the auxiliary unknown w. It is called a *resolvent* of the system. Except for certain solutions of a special nature, the study of the original system reduces to that of the single equation $R=0$. The author takes pains to show that when the λ's are properly chosen functions of x, no two distinct solu-

tions of the original system are associated with the same w. Other forms of (2) also lead to resolvents.

The application of the division algorithm requires that the polynomials be at least partially ordered relatively to one another. The scheme used is based on the lexicographical ordering, which is so prominent in Riquier's work. In order to avoid repetition (see pp. 3–6, 135, 142, 157–8) and to make the definitions clear and concise, placing a general discussion of this subject at the beginning of any treatment of systems would seem preferable to the author's way of introducing particular properties when and where they are needed. The essential features of the ordering are as follows. A set of symbols is called ordered if of any two symbols x, y in the set it is possible to say whether x precedes, coincides with, or follows y. These relations are conveniently denoted by $<$, $=$, $>$, and the terms positive and negative are useful in describing the relation of a symbol to 0. If the symbols x_i are taken from a set which is already ordered and which contains the difference of any two of its members, the symbols $x=(x_1, \cdots, x_n)$ can be ordered by saying that x precedes, coincides with, or follows y according as the first non-zero difference in the sequence $x_1-y_1, \cdots, x_n-y_n$ is negative, non-existent, or positive. The transitive law can readily be proved provided that two x_i's with the same sign have a sum with that sign. It is desirable at times to be able to compare $(x_1, \cdots, x_n)$ and $(y_1, \cdots, y_m)$, where $m<n$. Of the divers conventions that may be adopted we cite only two: $(y_1, \cdots, y_m)=(y_1, \cdots, y_m, 0, \cdots, 0)$ and $(y_1, \cdots, y_m)=(y_1, \cdots, y_m, \infty, \cdots, \infty)$, where ∞ is a symbol with the properties $\infty-y>0$, $y-\infty<0$, $\infty-\infty=0$, $\infty+\infty>0$. The transitive law holds for both of these. We must use the second convention to order ascending sets in Ritt's manner (p. 4).

The symbols useful in the present work are obtained by starting from the non-negative integers x_i, forming from them hypercomplex integers $(x_1, \cdots, x_n)$, from the latter integers $(y_1, \cdots, y_m)$, where each y_i is a hypercomplex integer, etc. The sets of symbols considered are such that the number of components in the parentheses is bounded. Any such set contains a minimum symbol. If it contains a maximum, it has only a finite number of distinct symbols (pp. 5, 6, 158). Each symbol represents a property of a polynomial, and the polynomial with the greater symbol has the property to a higher degree. Thus, the rank of a differential polynomial is the complex integer (c, g, s), where c is its class and g, s are respectively its order and degree in y_c. The rank in a particular unknown y is the complex integer (p, q), where p, q are respectively the order and degree in y. Something might be said in favor of sharpening the notion of rank so that any member of a basic set of a closed irreducible system would be algebraically irreducible. (See the author's repeated remark that a basic set can be chosen so as to have this property.)

It is interesting to compare the present method of elimination with Riquier's general method. In applying the latter, an equation has to be solved for one of the symbols in terms of the others, and the result substituted in a second equation. The fact that the eliminant is not in general expressed in finite terms gives rise to a difficulty, which is avoided by Ritt's method, whose eliminants appear as polynomials in the given functions and their derivatives. It is important, however, to realize that a difficulty of exactly the same nature may

arise from the necessity of recognizing whether a given polynomial in the coefficients is identically zero. This difficulty will entirely disappear only if the coefficients are restricted to be polynomials in the independent variables.

Every one will recognize that the author's method has obvious advantages over Riquier's, but some will not agree with him when he says (footnote, p. 161) "It is not easy to imagine systems other than linear systems for which Tresse's argument and result have a definite meaning." The theorem and process in question, although often called Tresse's, are more appropriately ascribed to Riquier, who, in addition to supplying the convergence proof omitted by Tresse, published his result fully a year earlier. Contrary to the quoted statement, it seems quite easy to write down non-linear systems for which one is forced to admit that Riquier's method has a very definite meaning. The system consisting of an equation solvable in terms of radicals and any number of equations deduced from it by differentiation furnishes an example. No content of the general system is lost, moreover, if the word "solve" in the description of the method is given the somewhat obvious interpretation "determine all the roots." In the reviewer's opinion Riquier's result has a definite meaning even in the general case, and the criticisms to which it is open may be classed as practical rather than theoretical. This, of course, does not mean that further refinement of the method is impossible or not to be desired.

In conclusion, the book is the outstanding contribution to the theory of systems since the publication of Riquier's treatise. Unlike the latter, it is concise and readable. Original, precise, clear, stimulating, it seems to embody what the colloquium series should. The author and Society are to be congratulated on their achievement.

J. M. THOMAS

SHORTER NOTICES

Traité du Calcul des Probabilités et de ses Applications. Edited by Émile Borel. Paris, Gauthier-Villars, 1933.

Vol. I, Section IV: *Les Principes de la Statistique Mathématique.* By R. Risser and C. E. Traynard. xi+388 pp.

Vol. III, Section IV: *Théorie Mathématique de L'Assurance Invalidité et de L'Assurance Nuptialité, Définitions et Relations Fondamentales.* By Henri Galbrun. 156 pp.

Vol. III, Section V: *Théorie Mathématique de L'Assurance Invalidité et L'Assurance Nuptialité, Calcul des Primes et des Réserves.* By Henri Galbrun. 183 pp.

This is an excellent, up-to-date treatment of mathematical statistics, covering an extensive range of well-selected topics. The earlier problems of statistics are given correct perspective; but the main purpose of the book seems to be to put the reader into contact with important recent developments, so far as space permits. Means and moments for one or more variables are handled with the aid of characteristic functions. For classifying the Pearson Types use is made of the following two criteria:

$$s = \{6\beta_2 - \beta_1 - 1\} \div \{3\beta_1 - 2\beta_2 + 6\};$$
$$p = \{4s^2(s+1)\} \div \{\beta_1(s+2)^2 + 16(s+1)\}.$$

Following the Bruns-Charlier series in Hermite polynomials, there appears Romanovsky's generalization with the substitution for the normal function of other frequency functions. Probable errors of statistical constants are discussed briefly with references to Karl Pearson, R. A. Fisher, and Rietz's *Handbook of Mathematical Statistics*; but small samples are not given detailed treatment. For drawing balls from urns, the schemes of Bernoulli, Poisson, Lexis, Borel, and Pólya are described. For the resolution of a frequency curve into components, Karl Pearson's method is given; and successive approximations are considered. This completes Part I.

Part II, devoted to correlation and covariation, draws from the works of Tschuprow, Galton, K. Pearson, Fechner, Cheysson, Norton, March, Keynes, Yule, Darmois, Lévy, Cantelli, Lexis, Bachelier, Edgeworth, Kneser, Myller-Lebedeff, Gram, Thiele, Charlier, Bruns, Galbrun, Guldberg, MacMahon, Student, Tchebycheff, Markoff, Bortkiewicz, Anderson, Steffensen, Sheppard, and Frisch. As part of this wealth of material may be mentioned the subject of probable values or mathematical expectation as developed by Tschuprow, the correlation surface in n dimensions as established by Pearson, also his chi-square surface, the Yule treatment of multiple and partial correlation, Anderson's variate difference method of dealing with correlation or covariation, Steffensen's modification of mean-square contingency, and Guldberg's difference equations for choosing the form of frequency functions.

In Section IV of the third volume a decidedly general treatment, based upon instantaneous rates, is given for insurance supplementary to life insurance. Disability is taken as representative of the entry into a new class where new mortality rates must be used; marriage as representative of the entry

into another class where new mortality rates are not needed. This marriage insurance seems to be intended primarily for children or minors who upon marriage are to receive the insurance as a species of dowry. In France, marriage insurance was authorized by law in May 1921. From $p(x, y)$, the probability that a man of age x will reach age y, usually written ${}_{y-x}p_x$, there is formed $q(x, y)dy = [-\partial p(x, y)/\partial y]dy$, the probability that a man of age x will die between ages y and $y+dy$. Likewise $w(x, y)dy$ is written as the probability that a man of age x will enter a new class—of the disabled or the married—between ages y and $y+dy$; and $p^b(\beta, \beta, y)$ is the probability that a man now of age β, who entered the new class b at age β, will survive to age y. With the original class designated by the superscript a, a typical equation is

$$p(x, x, y) = p^a(x, y) + \int_x^y w(x, \beta)p^b(\beta, \beta, y)d\beta.$$

This is the probability that a man now of age x and in class a will survive to age y in class a *or* in class b. The annual rate $W(x, x+1)$ of withdrawal from class a, by disability or by marriage, is the integral of $W(x, \alpha)d\alpha$ from x to $x+1$. After a great many pertinent distinctions have been made and numerous probabilities defined, suitable approximations for the required aggregates are made with the aid of the Newton interpolation formula. Then follows a critical note on the use of the product formula for compound probability. Finally, there are tables giving disability and marriage rates.

Section V is a continuation, in the same excellent style, of Section IV. From the complex probabilities evolved in Section IV, net premiums for a large variety of forms of insurance and annuities are deduced, and the corresponding reserves or policy values. The proper loading to be added to the net premium to form the office premium is treated at some length with the aid of the Tchebycheff inequality. Thus, for a large company, the percentage loading for safety is very small. The last chapter is devoted to a somewhat detailed discussion of the validity of the Tchebycheff inequality as utilized in the preceding chapters.

E. L. DODD

Höhere Mathematik. Teil IV: *Uebungsaufgaben mit Lösungen.* Heft II. By R. Rothe. Leipzig, B. G. Teubner, 1933. 53 pp.

This pamphlet gives interesting and instructive exercises (followed immediately by their solutions) on calculus topics included under the headings: (a) functions of two variables, their geometric representation, partial derivatives, and maxima and minima; (b) the differential geometry of plane curves, including properties of tangents and normals, order of contact, curvature, polar coordinate forms, asymptotes, and singular points. Further there are questions relating to complex numbers and the simpler functions of a complex variable.

T. H. HILDEBRANDT

Einführung in die moderne Atomphysik in allgemeinverständlicher Darstellung. By Arthur March. Leipzig, Barth, 1933. iv+115 pp.

The changes in our conception of the atom, initiated by the work of de Broglie, Schroedinger, and Heisenberg, have been so profound that interest in the innovations, and indeed knowledge of them, could not remain confined to a small circle of professional scientists. Accordingly, many attempts at explaining and publicising the new "truths" have been made, but few have been as successful as the book under review. The author does not undertake the ill-fated task of explaining quantum mechanics in words of one syllable. He addresses himself to the large group of individuals who have some knowledge of mathematics and the other exact sciences, and a healthy curiosity to know what is going on in physics.

If March's treatment of his subject were to be characterized by a single word, only the German "anschaulich" would be in place. Wave mechanics is introduced in the first part of the book, and a gradual transition is then made to statistical theory. Visual clarity is maintained throughout. The philosophical outlook is the one which has become orthodox among physicists today: theory describes experimental facts only; questions which can not be settled experimentally are meaningless, etc. Philosophers who are unwilling to embrace these doctrines may perhaps not find the book convincing.

Pedagogically, however, the book is excellent. It is cleverly illustrated with diagrams and figures. This reviewer has read it with considerable pleasure and finds in it many hints of value in teaching the subject.

HENRY MARGENAU

Le Problème de Cauchy et les Équations aux Dérivées Partielles Linéaires Hyperboliques. By J. Hadamard. Paris, Hermann, 1932. xiv+542 pp.

The present volume is a revised and considerably augmented translation of the author's classical *Lectures on Cauchy's Problem in Linear Partial Differential Equations* [Yale University Press, 1923, 324 pp.]. The main changes are introduced in Book IV, which contains, together with the older "method of descent" for the solution of Cauchy's problem, a direct method developed by the author in his memoirs of 1924. Another important change consists in the addition of three Appendices. Appendix 1 (*Forme invariantive donnée à la solution*) treats of a representation of solutions in invariant form, in conformity with the methods of the modern tensor calculus. Appendix 2 (*Notions sur la résolution du problème mixte*) contains an account of researches by various authors, including the author of the book, on the mixed problem for hyperbolic equations, which plays a rôle almost of the same importance as the Cauchy problem itself. Appendix 3 (*Détermination du problème dans le cas non linéaire*) treats of the uniqueness problem for a hyperbolic equation of general type and for the elliptic equation with analytic coefficients. Here the author follows mainly the interesting researches of H. Lewy. All these additions increase considerably the value of this great work so well known in this country.

J. D. TAMARKIN

NOTES

In addition to the members of the editorial board of the Transactions of the American Mathematical Society, the following persons have assisted the editors by refereeing papers offered for publication in the volume for 1933: R. P. Agnew, A. A. Albert, A. A. Bennett, A. S. Besicovitch, H. R. Brahana, H. T. Engstrom, L. M. Graves, T. H. Hildebrandt, E. Hopf, E. V. Huntington, M. H. Ingraham, G. A. Miller, S. Saks, W. Seidel, J. A. Shohat, H. S. Vandiver, G. E. Wahlin, J. H. M. Wedderburn. The editors appreciate this assistance and desire to give it this public recognition.

The Indian Mathematical Society has established a new quarterly journal called The Mathematics Student, to be conducted along lines similar to those of the American Mathematical Monthly (see this Bulletin, vol. 39 (1933, p. 857)). Hereafter, the Journal of the same Society will be issued as a quarterly, and will contain only original contributions of an advanced character. Information may be had from S. Mahadeva Aiyar, Presidency College, Madras, India.

The State University of Moscow has begun publication of a journal enentitled Abhandlungen aus dem Seminar für Vektor- und Tensoranalysis samt Anwendungen auf Geometrie, Mechanik und Physik, under the editorship of B. Kagan.

The American Mathematical Society has recently issued the sixteenth volume of its Colloquium Publications, *Algebraic Functions*, by G. A. Bliss. This was published with the cooperation of the National Research Council, with funds provided from its Revolving Book Fund. Copies may be obtained from the Society's office; the price is $3.00 (to members of the Society, $2.50).

The British Association for the Advancement of Science will meet at Aberdeen, September 5–12, 1934, under the presidency of Sir William Hardy. Professor H. M. Macdonald has been appointed sectional president of Section A (mathematical and physical sciences).

At the annual meeting of the London Mathematical Society, held November 16, 1933, the following officers and new members of the Council were elected: President, Professor G. N. Watson; Vice-Presidents, Professors A. C. Dixon, G. H. Hardy, G. F. J. Temple; Treasurer, Dr. A. E. Western; Librarian, Professor Harold Hilton; Secretaries, Mr. F. P. White, Mr. W. L. Ferrar; new members of the Council, Dr. M. L. Cartwright, Dr. G. B. Jeffery.

Professor E. H. Neville, of the University of Reading, has been elected president of the (British) Mathematical Association.

Professor A. H. Compton, of the University of Chicago, has been elected president of the American Physical Society.

Professor H. E. Slaught has been elected honorary president for life of the Mathematical Association of America. Professors A. A. Bennett and E. P. Lane have been elected vice-presidents, and Professors R. W. Brink, D. R. Curtiss, H. L. Rietz, and J. L. Walsh members of the Board of Trustees.

The Paris Academy of Sciences announces the award of the following prizes for 1933: The Francoeur prize to Paul Mentré, for his work in geometry; the Montyon prize in mechanics to René Thiry, for his work in the mechanics of fluids; the Poncelet prize in mechanics to E. B. de Fontviolant, for his work in mechanics; the Boileau prize to Adrien Foch, for his work in hydraulics; the Pierson-Perrin prize to Paul Langevin, for his work on the mechanical applications of piezo-electric quartz; the Lalande prize to Georges Prévost, for his tables of spherical functions and their integrals; the Valz prize to Henri Labrouste, for his methods of research in the periods of solar phenomena; the de Pontécoulant prize to David Beloritzky, for his work in celestial mechanics; the Henri de Parville prize in physics to Louis Leprince-Ringuet, for his work in atomic physics; the Henri de Parville prize for a scientific publication to Gustave Juvet, for his work entitled *La Structure des Nouvelles Théories Physiques*; the Bordin prize to Szolem Mandelbrojt, for his memoir *Sur l'Unicite des Séries de Fourier*; the Petit d'Ormoy prize in mathematical sciences to Arnaud Denjoy, for his mathematical work as a whole; the Estrade-Delcros prize to Ernest Vessiot, for his scientific work as a whole; the Saintour prize to Georges Giraud, for his work in partial differential equations and integral equations; the Charles Dupin prize to Bertrand Gambier, for his work in geometry; a prize from the Charles Frémont foundation to Léon Pomey, for his work in geometry and analysis.

The Pontifical Academy of Sciences (dei Nuovi Lincei) has awarded its international prize for a memoir on systems of solutions of differential equations (see this Bulletin, vol. 39, p. 19) to Professor G. D. Birkhoff, of Harvard University. It has also recommended that the essays offered by Professor L. Fantappiè, of Bologna, and Professors N. Kryloff and N. Bogoliouboff, of Kieff, be published by the Academy.

Professor A. Buhl, of the University of Toulouse, has been awarded the Cross of the Legion of Honor.

The Planck medal has been awarded to Dr. Werner Heisenberg, professor of theoretical physics at Leipzig.

The gold medal of the Royal Astronomical Society has been awarded to Professor Harlow Shapley, of Harvard University, for his studies of the structure and dimensions of the galactic system.

The Royal Society of London has awarded a Royal medal to Professor G. I. Taylor, for his distinguished theoretical and experimental researches in physics, geophysics, and aerodynamics.

The Bôcher prize of the American Mathematical Society has been awarded jointly to Professor Marston Morse, of Harvard University, for his memoir

entitled *The foundations of a theory of the calculus of variations in the large in m-space* (second paper), in volume 32 of the Society's Transactions, and to Professor Norbert Wiener, of the Massachusetts Institute of Technology, for his memoir entitled *Tauberian theorems*, in volume 33 of the Annals of Mathematics.

The National Academy of Sciences has awarded its Comstock prize to Professor P. W. Bridgman, of Harvard University, for his researches on the behavior of matter under high pressure.

The Edison medal of the American Institute of Electrical Engineers has been awarded to Professor A. E. Kennelly, of Harvard University, "for meritorious achievements in electrical science, electrical engineering, and the electrical arts, as exemplified by his contributions to the theory of electrical transmission and to the development of international electrical standards."

The gold medal of Villanova College has been awarded to the Abbé Georges Le Maitre, of the University of Louvain, visiting professor of astronomy at the Catholic University in Washington.

Professor A. Comessatti, of the University of Padova, has been elected a correspondent of the Academy of Sciences of Turin.

Professor F. Meyer, retired, of the University of Königsberg, has been elected a corresponding member of the Munich Academy of Sciences.

Professor René Thiry, of the University of Strasbourg, has been elected correspondent of the Paris Academy of Sciences in the section of mechanics.

Professor A. Tonolo, of the University of Padova, has been elected a correspondent of the Reale Istituto Veneto.

The University of Strasbourg has conferred an honorary doctorate on Professor Pieter Zeeman, of the University of Amsterdam.

On January 12, 1934, a dinner was given in honor of the seventieth birthdays of Professors E. B. Skinner, C. S. Slichter, and E. B. Van Vleck. This was also in honor of their long service to mathematics and to the University of Wisconsin where their aggregate term of teaching has been one hundred and fifteen years. Unfortunately Dean Slichter was ill and, although up to the afternoon of the dinner he had hoped to be there, it was impossible for him to be present. Professor M. H. Ingraham acted as toastmaster. Dean Sellery spoke first of the activities of all of the three men being honored, and particularly of Professor Skinner, and Professor Skinner replied to this address. Dr. Warren Weaver of the Rockefeller Foundation then spoke of the work and character of Dean Slichter, and President Frank, in response, read a reply that Dean Slichter had furnished that same afternoon. Professor G. D. Birkhoff of Harvard University then spoke of the research work and wide influence of Professor Van Vleck, and Professor Van Vleck replied. Ex-President Birge spoke of the work of all three of the men—and President Frank finally gave the thanks of the University to each for their long and valuable services.

Professor Albert Einstein has been elected to honorary membership in the American Astronomical Society.

Georgetown University has conferred the honorary degree of doctor of science on Dr. James Robertson, director of the Nautical Almanac Office of the United States Naval Observatory, in recognition of his work in theoretical astronomy.

Professor G. Andreoli, of the University of Catania, has been transferred to the University of Naples as professor of the calculus.

Dr. R. Caccioppoli, professor of algebraic analysis at the University of Padova, has been transferred to the professorship of the Galois theory of groups and algebraic equations at the University of Naples.

Dr. S. Cherubino, of the University of Naples, has been appointed associate professor of analytic and projective geometry at the University of Catania.

Professor H. G. Forder, head of the department of mathematics at Hymer's College, Hull, has been appointed professor of mathematics at Auckland College, New Zealand.

Dr. E. Gugino, of the University of Messina, has been promoted to an associate professorship.

Professor W. Heisenberg, of the University of Leipzig, has been appointed Scott lecturer at Cambridge University for the year 1934, and Professor G. von Hevesy, of the University of Freiburg i. Br., for the year 1935.

Dr. Lecaut has been appointed professor of rational and applied mechanics at the University of Rennes.

Dr. A. Masotti, of the Milan Technical School, has been promoted to an associate professorship of graphical statics.

Dr. G. D. Mattioli, of the University of Padova, has been appointed associate professor of rational mechanics at the University of Catania.

Dr. H. Milloux has been appointed professor of the infinitesimal calculus at the University of Bordeaux.

Professor Richard von Mises, director of the Institute for Applied Mathematics of the University of Berlin, and founder and for many years editor of the Zeitschrift für Angewandte Mathematik und Mechanik, has been appointed professor of pure and applied mathematics and director of the Mathematical Institute of the University of Istanbul. He will be succeeded as editor of the Zeitschrift by Professor Eric Trefftz, of Dresden.

The title of emeritus professor of eugenics has been conferred by the University of London on Dr. Karl Pearson on his retirement from the Galton chair of eugenics at University College.

Dr. W. Prager, formerly of Göttingen, has been appointed professor of mechanics at the University of Istanbul.

Dr. Fritz Reiche, professor of theoretical physics at the University of Breslau, has retired.

Dr. Erwin Schroedinger, professor of theoretical physics at the University of Berlin, has been appointed to a professorship in the same department at the University of Oxford.

Professor G. A. Bliss has been awarded the Martin A. Ryerson professorship at the University of Chicago, for his "brilliant and fruitful service to the University." Professor L. E. Dickson also holds a "distinguished service professorship" at that University.

Professor W. L. Bragg, of the University of Manchester, will be in residence at Cornell University from the beginning of February to the end of May, 1934, as visiting professor of chemistry.

Sir Arthur Stanley Eddington has been appointed Messenger lecturer at Cornell University for 1934. He will deliver a series of twelve lectures there on *New pathways of science* in April and May.

Dr. Howard Holston Alden, of Ohio State University, has been appointed head of the department of mathematics at the New Mexico Military Institute, Roswell, New Mexico.

Assistant Professor Max Morris has been promoted to an associate professorship at the Case School of Applied Science.

Harold Medway Martin, for forty-six years a member of the editorial staff of the English journal Engineering, and a mathematical physicist of distinction, died November 17, 1933.

Robert Hjalmar Mellin, retired professor of mathematics at the Helsingfors Technical School and for many years an editor of the Acta Mathematica, died April 5, 1933, at the age of seventy-eight.

Emile Meyerson, whose philosophical work has been of interest to mathematicians, died December 4, 1933.

The death is reported of Dr. Albert Wangerin, formerly professor of mathematics at the University of Halle.

Mr. John McDonnell, formerly of the Canadian Geodetic Survey of Ottawa, a member of the American Mathematical Society since 1914, died December 29, 1933.

Mr. E. R. Matthews, University fellow in mathematics at the University of California, Berkeley, is dead.

Dr. John Merrill Poor, professor of astronomy at Dartmouth College and head of the Shattuck Observatory, died December 11, 1933, at the age of sixty-two.

Professor P. L. Saurel, for many years head of the department of mathematics at the College of the City of New York, died January 21, 1934, at the age of sixty-three.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

104. Mr. Garrett Birkhoff: *Combinatorial relations in finite projective geometries.*

Under the restriction of finiteness, it is shown that B-lattices satisfying a (non-unique) complementarity relation are equivalent to direct joins of Boolean algebras and projective geometries. This has interpretations in projective geometry and group theory. (Received January 24, 1934.)

105. Mr. Aaron Fialkow: *General theorems on trajectories and lines of force.*

We generalize certain theorems of Kasner relative to the geometry of arbitrary fields of force (Science, June 24, 1932; p. 671). Consider the motion of a particle which starts from rest at a point where the line of force has contact of the nth order with its tangent. Kasner has shown that the trajectory has the same contact and that the ratio of the infinitesimal departures of the line of force and the trajectory from their common tangent is $2n+1$. We extend these results to the more general cases in which the contact between the line of force and its tangent is of any order, finite (integral, fractional or irrational) or infinite, as well as to cases in which no definite order of contact exists. In general, the ratio of departures approaches all values within a closed interval (rather than a single point), the interval varying with the order of contact. We also show that the limiting values of the ratio are uniquely determined by the direction of the force along any segment of the tangent line which includes the initial point. Kasner's theorems for the cases of non-zero speed and motion in a resisting medium are also generalized. (Received January 15, 1934.)

106. Professor E. V. Huntington: *Independent postulates related to C. I. Lewis's theory of strict implication.*

This paper presents an abstract mathematical system of which Lewis's system (as verbally formulated) may be regarded as an instance. *Base*: a class K (interpretable as "propositions"); two subclasses Q and D (interpretable as "false" and "necessarily false"); $a \times b$ (interpretable as "a and b"); a' (interpretable as the "contradictory" of a); and $a = b$, meaning that a and b are interchangeable for the purposes of the system. *Definitions*: (I) (a in P) means (a in K and a not in Q). (II) (a in A) means (a' in D). (III) ($a < b$) means (ab' in

D). *Postulates*: (1) If a in K and b in K, then ab in K. (2) If a in K, then a' in K. (3) $ab=ba$. (4) $(ab)c=a(bc)$. (5) $(a'b')'(a'b)'=a$. (6) If a in Q, then a in K. (7) If a in D, then a on Q. (8) If a in D and b in K, then ab in D. (9) If a in P and b in P, then ab in P. (10) If a in K and a' in Q, then a in P. (11) If a in A and b in A, then ab in A. (12) There exists at least one element b in K such that b not in D and B not in A. (13, optional) If a in D and b in D, then $a=b$. (When (13) is added, (11) becomes redundant.) Points of divergence between ordinary mathematical methods and certain procedures characteristic of the logistic method are indicated. (Received January 29, 1934.)

107. Professor Louis Weisner: *Criteria for the irreducibility of polynomials.*

Let $A(x)$ be a polynomial of degree n with integral coefficients, the coefficient of x^k being a_{n-k}, and let $a_n=\pm kp^m$, where p is a prime which does not divide a_{n-1} if $m>1$. Let L and M be lower and upper bounds of the absolute values of the roots of $A(x)=0$. The author proves that if $A(x)$ is reducible in the field of rational numbers and $L\geqq 1$, then $k\geqq L$, while if $M\geqq 1$, then $p^m\leqq |a_0|M^{n-1}$. It is possible to construct large classes of polynomials which satisfy the hypothesis, but which violate one of the inequalities of the conclusion, and which are therefore irreducible. Further, by effecting a linear transformation on the polynomial, a theorem is deduced which may enable one to infer the irreducibility of a polynomial from its table of values. (Received December 30, 1933.)

108. Professor Antoni Zygmund: *Some points in the theory of trigonometric and power series.*

This paper consists of six notes treating various related topics. In the first note it is proved that if the partial sums of the Fourier series of $f(x)$ are bounded below by a negative integrable function, then almost everywhere $f(x)$ is equal to the arithmetic mean of $\limsup s_n$ and $\liminf s_n$. The second note is concerned with the exponent of convergence of the sequence of Fourier coefficients of a function which is of bounded variation and satisfies a Lipschitz condition of positive order. In the third note there is found a simple proof of a theorem of Kolmogoroff concerning the conjugate of a Fourier series. The fourth note contains a new proof of the classical theorem of Fejér-F. Riesz and of some of its extensions. The fifth note deals with the extension of some results of Hardy and Littlewood concerning the theory of fractional integrals, while the sixth note contains extensions of some results of the same authors in the theory of Fourier constants. (Received January 24, 1934.)

109. Professor E. P. Lane: *The moving trihedron.*

A classical method of studying the metric differential geometry of curves and surfaces in three-dimensional space is based upon the use of a moving trihedron. Professor G. A. Bliss constructed several years ago a theory of the moving trihedron in the theory of curves, and used it in his lectures, but has never published it. He used the moving trihedron whose edges are the tangent, principal normal, and binormal at a point of a curve, and based his theory on certain *recursion formulas*, which are the novel feature of the treatment. They

give the components, in the moving coordinate system, of the derivatives of any order of the vector defining a curve whose points are in one-to-one correspondence with the points of the given curve. The present paper outlines the theory of curves and then extends it to surfaces. The author uses the moving trihedron whose edges are the tangents of the lines of curvature and the normal at a point of a surface. Recursion formulas are again employed. The six fundamental coefficients of a surface, whose points are in one-to-one correspondence with those of a given surface, are calculated in terms of the components of the derivatives of the vector defining the former surface, referred to the moving trihedron of the latter. The results are capable of wide application. (Received January 8, 1934.)

110. Professor H. R. Brahana: *Irreducible quartic congruences.*

The irreducible quartics belonging to the modular field determined by a prime p constitute $(p-1)/2$ sets of conjugates under the group of linear fractional transformations with coefficients also in the modular field. Each set is characterized by the cross ratio of the roots of the quartic; each set is also characterized by the value of the absolute invariant of the quartic. (Received January 19, 1934.)

111. Professor R. D. Carmichael: *A general expansion theorem with applications to certain linear integral equations of infinite order.*

A general theorem is given for the expansion of functions $f(x)$, analytic at $x=0$, in the form $f(x)=\sum_{\nu=0}^{\infty} c_\nu \{x^\nu/\lambda_0\lambda_1 \cdots \lambda_\nu\} \{1+a_1(x)\cdot x/\lambda_{\nu+1}+a_2(x) \cdot x^2/(\lambda_{\nu+1}\lambda_{\nu+2})+ \cdots \}$, where $\lambda_0, \lambda_1, \cdots$ is a given non-decreasing sequence of positive constants with the limit infinity and where $a_1(x), a_2(x), \cdots$ is a sequence of given analytic functions subject to suitable restrictions. The expansion theorem is applied in the almost immediate derivation of existence theorems for linear differential equations, for certain linear integral equations of infinite order, and for certain integro-differential equations. (Received January 27, 1934.)

112. Professors Tibor Radó and Lincoln LaPaz: *On a converse of the Kneser transversality theorem.*

In the special case in which transversality is orthogonality, E. Kasner for euclidean 3-space (Princeton Colloquium Lectures, 1909, p. 47) and J. Lipke for Riemannian n-space (Transactions of this Society, vol. 13 (1912), p. 77) have proved an important converse of the Kneser transversality theorem. J. A. Schouten (Nieuw Archief voor Wiskunde (1927), p. 97) has given a simplified proof of Lipke's results. Independently W. Blaschke (Nieuw Archief voor Wiskunde (1927), p. 202) and J. Douglas (Transactions of this Society, vol. 29 (1927), p. 401) have given proofs of a converse of Kneser's theorem for the case of a general transversality. The purpose of the present paper is to give a proof which emphasizes the very simple geometrical background of the theorem in question. The proof is based on a congruence relation obtained by applying to finite arcs a construction employed by Blaschke for infinitesimal arcs. (Received February 6, 1934.)

113. Professor T. R. Hollcroft: *Webs of quadric hypersurfaces in r dimensions.*

The properties of the web of quadric hypersurfaces in S_r are obtained by means of the (1, 1) correspondence existing between the quadric hypersurfaces of the web and the planes of an S_3. For $r \geqq 4$, the web contains no completely degenerate quadrics. For all values of r, however, the web contains $r(r+1)(r+2)$ pencils of quadric hypersurfaces that have a line in common. (Received March 9, 1934.)

114. Professor Virgil Snyder and Dr. J. M. Clarkson: *An involutorial line transformation determined by a bilinear congruence of twisted elliptic quartics.*

Given a fixed plane γ and two non-singular pencils of quadric surfaces $H_1-\alpha H_2=0$, $K_1-\beta K_2=0$, having as basis curves the twisted quartics α, β respectively. The curve $C_4(\alpha, \beta)$ of intersection of one quadric of each pencil is determined when α, β are given. As the parameters α, β vary independently, $C_4(\alpha, \beta)$ describes a bilinear congruence of twisted quartics of first species. An arbitrary line (y) of space is bisecant to just one C_4 of the congruence and meets γ in a point P. Through P there exists one other bisecant (x) of the same C_4. The line transformation $(y)\sim(x)$ is discussed. (Received March 2, 1934.)

115. Dr. A. B. Brown and Mr. Morris Halperin: *On certain area-preserving maps.*

It is proved that any simply-connected region of finite area in the plane can be mapped in a continuous one-to-one manner with preservation of area on a circle-interior. A characterization is given in terms of a set of nested curves and a curve crossing them, which are mapped on concentric circles and a radius respectively. This covers among others all maps which when expressed in cartesian coordinates are given by functions having continuous first partial derivatives. Some of the results are extended to curved surfaces and three-dimensional regions, with equations of the transformations given. The method permits extending the results to regions with infinite content. (Received February 7, 1934.)

116. Dr. S. F. Barber (National Research Fellow) and Dr. Jacob Yerushalmy: *Exceptional curves of the first kind.* Preliminary report.

Heretofore in the theory of birational transformations, the cases of irreducible exceptional curves or those with at most two components have been considered. This paper treats the general type of exceptional curve, $L^{(1)}=\sum_{i=1}^{s} k_i^{(1)} L_i$, $k_i^{(1)} \geqq 1$. Associated with $L^{(1)}$ are found other exceptional curves $L^{(r)}=\sum_{i=1}^{s} k_i^{(r)} L_i$. In the correspondence between $L^{(1)}$ and O_i, the position of these points is described by a branch of minimum order passing through them. It has been found that the branch of minimum order passes through O_i $(i=1, \cdots, t)$ with multiplicity $k_i^{(1)}$, $t=1, \cdots, s$. The authors are now

investigating the connection with the characteristic exponents in the Puiseux expansion and the relation of the intersection numbers of the components with satellite and free points. (Received March 9, 1934.)

117. Dr. L. M. Blumenthal (National Research Fellow): *The metric characterization of the n-dimensional spherical space.*

The n-dimensional spherical space, $S_{n,r}$, of radius r, is the surface of an $(n+1)$-dimensional sphere in a euclidean space of $n+1$ dimensions, with the distance between two points defined as the geodesic distance. This paper defines an abstract space S_r as a metric space the elements of which satisfy the following two postulates: Postulate 1. The sum of the three distances determined by any three points of the space does not exceed $2\pi r$, $r>0$. Postulate 2. If a quadruple of points of the space contains a triple that is isometric with three points of $S_{1,r}$, the quadruple can be isometrically imbedded in $S_{2,r}$. The space is said to be diametrized if for each point p the space contains a point $\bar{p}$ such that $p\bar{p}=\pi r$. This paper investigates the properties of S_r. The principal results obtained are stated in the two following theorems. Theorem 1. *If S_r is convex, complete, and diametrized, then any $n+1$ points of S_r can be isometrically imbedded in $S_{n,r}$.* Theorem 2. *If S_r is convex, complete, separable, and diametrized, then S_r may be transformed by an isometric transformation into $S_{n,r}$ or H_r, where H_r is the spherical analogue of Hilbert space.* (Received March 5, 1934.)

118. Dr. D. S. Nathan (National Research Fellow): *One-parameter groups of transformations in abstract vector spaces.*

A transformation T in a complete linear metric space $\mathfrak{B}$ satisfying the inequality $\|Tf-Tg\|\leqq C\|f-g\|$, $C\geqq 0$, for every pair of elements f and g in $\mathfrak{B}$, generates a one-parameter continuous group of transformations in $\mathfrak{B}$. When T is a bounded linear transformation, the generated group is the group of bounded linear transformations e^{tT}, $-\infty<t<\infty$. Conversely, a subset, in a certain neighborhood of the identity transformation, of a given one-parameter continuous group of bounded linear transformations A_t in $\mathfrak{B}$ with the law of composition $A_{t_1}A_{t_2}=A_{t_1+t_2}$ is generated by the bounded linear transformation $\log(A_t/t)$, defined in $\mathfrak{B}$. (Received March 8, 1934.)

119. Mr. George Comenetz: *Families of ∞^3 conics which are curvature trajectories.*

Curvature trajectories are a special type of family of ∞^3 plane curves, constructed according to the following geometrical rule. We take a family of ∞^2 curves arbitrarily as a base. A curvature trajectory of these is a curve drawn through the base family in such a way as to have its curvature at each point equal to c times the curvature of the member of the base family which it touches at that point, c remaining constant on any one trajectory. This construction is projectively invariant. We show that the only families of curvature trajectories which consist solely of conics are the following. (A) The projections, from a fixed center on to a fixed plane, of all the conics drawn on a quadric surface. (B) The two types of families of conics which are dynamical

trajectories, determined by Darboux: conics with a common focus, of the Newtonian field, with their projections; and conics with a common center, of the elastic field, with their projections. (See Kasner, *Dynamical trajectories and curvature trajectories*, to appear shortly in this Bulletin.) (Received March 8, 1934.)

120. Professor Edward Kasner: *Mixed groups connected with polygons.*

This paper generalizes a theory of mid-point polygons and central symmetry published in the American Mathematical Monthly, 1903. Centroidal polygons are introduced and multi-point transformations present themselves. The generalized symmetries generate a mixed group of $r\infty^{2(r-1)}$ transformations operating on sets of $r-1$ points. A joint paper with several of the author's students will appear in Scripta Mathematica. (Received March 5, 1934.)

121. Professor R. L. Wilder: *Generalized closed manifolds in n-space.*

We define a generalized closed n-dimensional manifold (=g.c. n-m.) as an n-dimensional compact metric space M such that (1) $p^n(M)=1$, but for any proper closed subset F of M, $p^n(F)=0$; (2) given $P\subset M$, there is an $\epsilon>0$ such that if $\gamma^i\subset S(P,\epsilon)$, $1\leqq i\leqq n-1$, then $\gamma^i\sim 0$ in M; (3) given $P\subset M$ and $\epsilon>0$, there exist positive numbers δ and η, $\epsilon>\delta>\eta$, such that if $\gamma^i(0\leqq i\leqq n-2)$ is a cycle of $F(P,\delta)$, then $\gamma^i\sim 0$ in $S(P,\epsilon)-S(P,\eta)$; and if γ^{n-1} is a cycle of $F(P,\delta)$, then $\gamma^{n-1}\sim 0$ in $M-S(P,\eta)$. It is shown that in order that a set M in E_n should be a g.c. $(n-1)$-m., it is necessary and sufficient that it be the boundary of a bounded, simply $(n-1)$-connected, uniformly locally i-connected (=u.l. i-c.) domain $(0\leqq i\leqq n-2)$. This condition may be replaced by the condition that M be the common boundary of two u.l. 0-c. domains D_1 and D_2 one of which is u.l. i-c. for $1\leqq i\leqq n-3$ and $p^{n-2}(E_n-M)$ finite. A new duality is established: $p^i(D_1)=p^{n-i-1}(D_2)$, $1\leqq i\leqq n-2$, and it follows that M satisfies the Poincaré duality relation. M itself is locally i-connected for $0\leqq i\leqq n-2$, and all its Betti numbers are finite. (In E_3, M is a closed manifold in the classical sense.) If M is a set satisfying only conditions (2) and (3) (with n replaced by $n-1$), then its complementary domains all have boundaries that are g.c. $(n-1)$-m.'s. (Received March 9, 1934.)

122. Mr. Saunders MacLane: *A systematic theory of mathematical symbolism.*

The development of the Hilbert "proof theory" and of the "semantic" logic of the Polish school have emphasized the importance of the analysis of mathematical symbolism. This analysis should rank as one of the basic disciplines in the foundations of mathematics on a par with the theory of classes and the theory of propositions. This paper presents a systematic analysis of symbolism in a simple and general form, not restricted to the use of any particular symbols for particular purposes. It is shown that any symbolic expression can be broken up into a number of "irreducible" expressions, such as $a+b$, $x=y$, $p\supset q$, and

the like. The chief operation in the manipulations of symbols is that of *replacing* one of the variables in such an irreducible expression by some already constructed expression E, thus giving a more complex expression in which E is a *component*. Using this operation, we can define the relation which makes one expression A a *case* of a second expression B, and the converse relation in which B is a *form* of A. These two relations are of great importance in the application of the theory of symbolism to logic. (Received March 9, 1934.)

123. Dr. Deane Montgomery (National Research Fellow): *A metrical property of point set transformations.*

The following theorem is proved: If E is a conditionally compact arc-wise connected metric space and if T is any continuous one-one transformation of E into itself, then there are two distinct points of E whose distance remains invariant under T. This theorem applies in particular to open or closed spheres in n-dimensional space and to spheres in function space. (Received February 24, 1934.)

124. Dr. G. T. Whyburn: *Non-alternating transformations.*

If A and B are compact and metric, a continuous transformation $T(A)=B$ is said to be non-alternating if for no two points x and y of B does the set $T^{-1}(x)$ separate two points of $T^{-1}(y)$ in A. A study is made of such transformations along with a similar investigation of those transformations in which the sets $T^{-1}(x)$ are all connected, the latter being equivalent to upper semi-continuous decompositions of A into continua. Among other things it is shown that if A is locally connected, each true cyclic element of B is the image under some non-alternating transformation of a cyclic element in A. Various applications of the results to the cases of spheres, circles, cactoids, and boundary curves are obtained. (Received March 3, 1934.)

125. Dr. John Williamson: *The covariants of two quadratic forms in n variables.*

Let f and g be two quadratic forms in the n variables $x_1, x_2, \cdots, x_n$, with matrices A and R respectively. It is known that, if ϕ and γ are the tangential forms of f and g, the coefficients of the powers of λ in the point form of the pencil $\phi+\lambda\gamma$ are projective covariants of f and g and, together with their jacobian J, form a complete system of covariants. We consider another set of n quadratic covariants, namely those whose matrices are R, A, ASA, $A(SA)^2, \cdots, A(SA)^{n-2}$, where S is the matrix adjugate to R. By means of this system we are able to show how the composition of certain of the irreducible concomitants of f and of g depends on the elementary divisors of $A+\lambda R$. In particular we show that, if R is non-singular and the elementary divisors of $A+\lambda R$ are $(\lambda+\lambda_i)^{e_i}$, $i=1, 2, \cdots, t$; $e_1+e_2+\cdots+e_t=n$, $\lambda_i\neq\lambda_j$, if $i\neq j$, then the jacobian $J=Ik_1^{e_1}k_2^{e_2}\cdots k_t^{e_t}$, where I is an invariant and $k_1, k_2, \cdots, k_t$ are t linearly independent linear forms in $x_1, x_2, \cdots, x_n$. (Received March 2, 1934.)

126. Mrs. Grace M. Hopper and Professor Oystein Ore: *New types of irreducibility criteria.*

Instead of the open Dumas polygon usually applied in the deduction of irreducibility criteria, a closed convex polygon is introduced, depending both on the size and the divisibility properties of the coefficients. For this polygon an approximate multiplication theorem holds and this may be used to deduce irreducibility criteria depending on the size of the coefficients. (Received March 7, 1934.)

127. Mr. Saunders MacLane: *On the representation of finite dual groups.*

Abstract dual groups, as discussed by Dedekind, F. Klein, and G. Birkhoff, include many common systems involving two operations, such as the operations of least common multiples and greatest common divisor or the operations (for classes) of meet and join. In particular, a dual group is of *ideal type* when these operations satisfy, beside the usual axioms, both possible distributive laws. The chief result of the paper is the determination of all possible finite dual groups of this sort. This is done by first constructing a *normal* point-set representation of the group and then setting up a *basis* for this representation. The elements of the group can be uniquely represented in terms of this basis, and thus we obtain a finite analogue to some of the decomposition theorems of ideal theory. The use of a basis also gives a necessary and sufficient condition for the isomorphism of two groups and an analysis of the *free* groups on n elements. Finally, certain relations of inclusion in the basis can be defined. For any preassigned basis with given inclusion relations there must exist a corresponding group. (Received March 9, 1934.)

128. Dr. Ralph Hull (National Research Fellow): *A canonical generation of rational cyclic algebras of odd prime degree.*

Every rational cyclic algebra A of odd prime degree n, that is, every cyclic algebra over the field R of all rational numbers of order n^2 over that field, is shown to have a cyclic generation $A=(\sigma, Z_p, S)$ where $\sigma=1$ if A is a total metric algebra, $\sigma=q_1, \cdots, q_r$ is a product of $r(\geqq 2)$ distinct primes q_i if A is a division algebra. The field Z_p is cyclic over R of degree n over R with a prime conductor $p\equiv 1 \pmod{n}$, that is, Z_p is the class field for the division of rational numbers prime to p into n classes consisting of the class of n-ic residues modulo p and $n-1$ classes of n-ic non-residues modulo p. Each prime q_i is distinct from p and is an n-ic non-residue modulo p while the q_i are distributed in the $n-1$ classes of n-ic non-residues modulo p in such a way that σ is itself an n-ic residue modulo p. Finally, S is a generating automorphism of the cyclic group of automorphisms of Z_p. The integer σ is uniquely determined by A, the primes q_i being precisely those primes for which the A-invariants ν_{q_i} (see Hasse, Transactions of this Society, January, 1932) are different from zero. For a given σ there exist infinitely many primes p with the properties described. (Received February 12, 1934.)

129. Dr. D. H. Lehmer: *An extended theory of Bernoulli and Euler numbers.*

It is customary to associate with the numbers B_n, E_n, of Bernoulli and Euler, two other sequences of numbers G_n, R_n which are in fact the coefficients of the Maclaurin expansion of the tangent and cosecant. It is the purpose of this paper to discuss the properties of a set of m^2 sequences of rational numbers, which coincide for $m=2$ with the 4 already mentioned, and which are the coefficients of the Maclaurin expansions of the m reciprocals and the $m(m-1)$ ratios of the m Olivier functions of order m. The kth Olivier function of order m is a power series whose terms are obtained from the series for e^x by selecting every mth term beginning with the kth term $(0 \leqq k < m)$. Most of the theory already developed for B_n, E_n, R_n, and G_n can be extended to the general case of m^2 sequences. We have, for instance, generalizations of the fundamental recurrence formulas, the divisibility properties, periodicity with respect to a modulus, the von Staudt-Clausen theorem, etc. In order to express the terms of these sequences by infinite series, it is necessary to generalize the ordinary Dirichlet series to a sum which extends over the roots of the Olivier functions. Except for $n=2$, 4, and 6 these roots are not distributed uniformly. (Received March 9, 1934.)

130. Dr. Selby Robinson: *Pseudo-transitivity in finite and infinite groups.*

A group is pseudo-k-fold transitive if it has elements that transform any *unordered* set of k points into any other, and quasi-k-fold transitive if it has this property for every $r \leqq k$. It is shown in this paper that a quasi-k-fold transitive group G, of sufficiently high finite degree, is $k-2$ times transitive. Except when $5 \leqq k \leqq 8$, G is $k-1$ times transitive; while if $k \leqq 10$ and composite, G is k times transitive. There are a number of theorems in the paper that apply to both finite and infinite groups. For example: a necessary and sufficient condition that a quasi-4-fold transitive group be doubly transitive is that it have a transformation which leaves a set of three points invariant but not pointwise invariant. (Received March 7, 1934.)

131. Professor A. A. Albert: *A solution of the principal problem in the theory of Riemann matrices.*

The principal problem in the theory of Riemann matrices is that of determining the multiplication algebra D of any pure Riemann matrix. Algebra D is a normal division algebra over its centrum K which is either a total real field or a pure imaginary quadratic extension of a total real field. The author solved the problem in the first case in an earlier paper. He now proves that in the second case the algebra D is a certain type of cyclic algebra of degree n over K, and that conversely there exist division algebras D of the given type for any n and pure Riemann matrices with D as multiplication algebra. (Received March 7, 1934.)

132. Professor A. A. Albert: *A matrix proof of the Poincaré theorem on impure Riemann matrices.*

The best proof of the Poincaré theorem on impure Riemann matrices in the literature is probably the very complicated projective geometric proof of G. Scorza (Palermo Rendiconti, 1916). The purpose of the present paper is to provide a purely algebraic proof of this important theorem as well as of the theory of the reduction of an impure Riemann matrix to pure components. The proofs given are far more elegant than any in the literature and provide the first purely algebraic treatment of the subject of *impure Riemann matrices.* (Received March 7, 1934.)

133. Professor A. A. Albert: *Cyclic fields of degree p^n over F of characteristic p.*

E. Artin and O. Schreier have given a determination of all cyclic fields of degree p and p^2 over a field F of characteristic p. In the present paper the author solves the corresponding problem in the case of degree p^n, the general case. (Received March 7, 1934.)

134. Professor P. A. Caris: *Integral solutions of $P^4+4Q^4=R^4+4S^4$.*

Numerical solutions of the diophantine equation $P^4+kQ^4=R^4+kS^4$, where k is given, have been found by several writers for various values of k. But the only case in which parametric solutions have been obtained hitherto seems to be the case $k=1$ for which Euler found a two-parameter solution involving polynomials of the seventh degree. In the present paper a parametric solution is found for the case $k=4$. The method is different from that of Euler. The result is $(a^3-4a^2b-2ab^2-4b^3)^4+4(a^3+a^2b+4ab^2-2b^3)^4=(a^3+4a^2b-2ab^2+4b^3)^4+4(a^3-a^2b+4ab^2+2b^3)^4$. (Received March 2, 1934.)

135. Dr. J. L. Dorroh: *Concerning the direct product of algebras.*

This paper gives the conditions under which two algebras which do not possess a finite basis have a direct product. (Received March 6, 1934.)

136. Mr. Garrett Birkhoff: *On the lattice theory of ideals.*

A uniqueness theorem on the direct decomposition of finite lattices is proved, together with a necessary and sufficient condition for the existence of such a decomposition in the case of B-lattices. On this basis the combinatorial relations of "einartig" ideals in a commutative ring with unity are discussed, and a criterion that the ideals should combine like point sets is derived. (Received February 7, 1934.)

137. Professor Edward Fleisher: *On Euler squares.*

In the Annals of Mathematics, vol. 23, No. 3, March, 1922, H. F. MacNeish gave a proof of the impossibility of the construction of Euler squares of order

n, degree 2, index $(n, 2)$ (Graeco-Latin squares) when $n \equiv 2 \pmod 4$ for the case in which the square, interpreted as a set of 2-circuits, lies on a manifold or on a set of manifolds in which each vertex is taken with the same multiplicity. All known Graeco-Latin squares have this property but no proof that the condition is necessary appears in the literature. The present author considers Euler squares of index $(n, 2)$ which are formed by operating on either the first row or the first column of dyads by means of a regular group of substitutions. This is true of all known squares. He shows that a square of index (rs, ν) formed by the composition of squares of indices (r, ν) and (s, ν) is based on a group which is the direct product of the groups of the component squares. He proves that no Euler square of order n, degree 2, based on a group, can be formed when $n \equiv 2 \pmod 4$. He also gives a linear-graph equivalent of the theorem that no Euler squares of index $(n, 3)$ can be constructed when $n \equiv 3 \pmod 9$. (Received February 12, 1934.)

138. Dr. R. D. James (National Research Fellow): *On the expansion coefficients of the functions $u/sn\ u$ and $u^2/sn^2 u$.*

The values of certain coefficients in the expansion of $u/\mathrm{sn}\ u$ and $u^2/\mathrm{sn}^2\ u$ were stated without proof by Hermite (Oeuvres, vol. 3, pp. 236–237), and no proof seems to have been published. In this note a proof is given following the method of Gruder (Wiener Sitzungsberichte, IIa, vol. 126 (1917)). The proof depends on several recursion formulas for the Bernoulli numbers. (Received February 28, 1934.)

139. Mr. Rufus Oldenburger: *Transposition of indices in multiple-labeled determinants.*

An ordinary determinant with more than one index associated with its rows or columns is called a *multiple-labeled determinant*. It is shown in this paper how these determinants arise under non-singular linear transformations on multilinear forms, where they possess relative invariant properties, and if sufficiently specialized factor into determinants of lower orders. The effect of transpositions of indices on the value of a multiple-labeled determinant, its minors, and cofactors, is then determined. The latter problem resolves itself into a study of the evenness or oddness of the substitutions necessary to bring a multiple-labeled array of numbers of the type $11, 12, \cdots, 1n, 21, \cdots, 2n, \cdots, m1, \cdots, mn$ into the array $11, 21, \cdots, m1, 12, \cdots, m2, \cdots, 1n, \cdots, mn$. Finally, it is shown how the matrices associated with multiple-labeled determinants are related to matrices used by Dirac and Weyl. (Received March 3, 1934.)

140. Mr. Rufus Oldenburger: *Factorization of hyperspace matrices into non-singular matrices.*

Necessary and sufficient conditions for a matrix $A = (a_{i_1 \cdots i_p})$ to factor in the form $(a_{\alpha i_p}{}^{(1)} \cdots a_{\alpha i_p}{}^{(p)})$, where $(a_{\alpha i_1}{}^{(1)}), \cdots, (a_{\alpha i_p}{}^{(p)})$ are matrices non-singular on the index α (rows), are given by the author in another paper. In the present paper necessary and sufficient conditions are given for the factorization

of the matrices $A=(a_{ij_1}\cdots_{j_p})$ and $A=(a_{ij_1k_1}\cdots_{k_p})$. The problem resolves itself into a problem in the equivalence of n-tuples of matrices. (Received March 3, 1934.)

141. Dr. M. R. Hestenes (National Research Fellow): *Sufficient conditions for the problem of Bolza in the calculus of variations.* II.

Until recently sufficient conditions for a minimum in the problem of Bolza have been given only for arcs which are not only normal relative to the end conditions but also normal on every subinterval. In a first paper (abstract 39–7–213) the author has shown that by a suitable modification of the condition of Mayer a set of sufficient conditions is obtained involving the assumption of normality relative to the end conditions but not the assumption of normality on the subintervals. In the present paper it is shown that the assumption of normality relative to the end conditions also is not needed. We merely need the assumption that there exists a set of multipliers of the form $\lambda_0=1$, $\lambda_\beta(x)$ with which the arc under consideration satisfies the usual sufficient conditions except possibly for a suitable modification of the condition of Mayer. In the normal case these sufficient conditions are identical with the known ones. (Received February 23, 1934.)

142. Mr. F. J. Murray: *Linear transformations between distinct Hilbert spaces.*

The usual theory of linear transformations in Hilbert space is first generalized to the theory of linear transformations between Hilbert spaces. For T continuous and linear we obtain a method for the computational determination of the general solution of the equation $Tf=g$. For T closed and linear, we determine the set of all closed linear manifolds $\mathfrak{M}$ such that if $f\in\mathfrak{D}$, then $f=f_1+f_2$, where $f_1\in\mathfrak{M}\cdot\mathfrak{D}$, $f_2\in\mathfrak{D}\cdot\mathfrak{h}\ominus\mathfrak{M}$ and Tf_1 is orthogonal to Tf_2. The theory of such manifolds for a closed linear transformation is quite analogous to the theory of manifolds which reduce a self-adjoint transformation. By their use we develop methods which compensate, in the study of possible inverses of a closed linear transformation, for the lack of compactness of Hilbert space. (Recived March 5, 1934.)

143. Dr. A. H. Smith: *Summability of the derived conjugate Fourier series.* Preliminary report.

B. N. Prasad (Journal de Mathématiques Pures et Appliquées, (9), vol. 11 (1932), p. 178) has proved a theorem concerning the Cesàro summability of the conjugate Fourier series. Using the Bosanquet-Linfoot method of summation (Journal of the London Mathematical Society, vol. 6 (1931), pp. 117–126), we extend in a similar manner the results of an earlier paper (this Bulletin, abstract 39–9–262) concerning the summability of the rth, $r\geqq 0$, derived conjugate Fourier series of a function $f(x)\subset\mathfrak{L}$. A more general result than Prasad's is obtained as a special case, namely when $r=0$, since the Bosanquet-Linfoot method of summation is weaker than that of Cesàro. (Received March 7, 1934.)

144. Dr. Deane Montgomery (National Research Fellow): *Properties of functions of two variables.*

If p is any property of functions of one variable and if $f(x, y)$ is any function of two variables, it is an interesting problem to determine the nature of the set of $\bar{x}$'s for which $f(\bar{x}, y)$ has the property p. This problem is completely solved in this paper when $f(x, y)$ is in the Baire classification and when p is any one of several properties among which are Riemann integrability, differentiability, and the property of having bounded variation. (Received February 24, 1934.)

145. Dr. H. W. Raudenbush: *Hypertranscendental extensions of partial differential fields.*

Differential fields, as defined by R. Baer, have a role in the abstract or formal theory of ordinary algebraic differential equations analogous to the role of field in abstract algebra. With an abstract theory of partial differential equations in view, the analogous concept of partial differential field is defined. The analogues to Steinitz' theorems on transcendental extensions of fields, obtained previously for differential fields, are carried over to partial differential fields. (Received March 5, 1934.)

146. Dr. J. L. Doob (National Research Fellow): *Probability distributions and statistics.*

Let $\mathbf{x}$ be a chance variable which can take on any real value and let $F(x)$ be the probability that $\mathbf{x}<x$. For n repeated trials a new set-up is needed consisting of n-dimensional euclidean space with a metric determined by $F(x)$. It is shown that there is a space of infinitely many dimensions which serves for an experiment with an arbitrary number of trials. This space is the space whose points are infinite sequences: $\cdots x_{-1}, x_0, x_1, \cdots$ where x_i is any real number, with a probability function (i.e., a completely additive non-negative set function with value 1 on the whole space) defined on it. By the use of the ergodic theorem of Birkhoff a generalization of the Law of Large Numbers is obtained: namely, that a necessary and sufficient condition that in a repeated trial $\lim_{n\to\infty} \sum_1^n (x_i/n) = c$ (where c is some constant) with probability 1, is that the expectation of the chance variable exists, and if this is so, c is this expectation. (This result was announced without proof by Khintchine.) These results are applied to prove rigorously certain well known theorems of R. A. Fisher used in statistics, concerning the principle of maximum likelihood. (Received March 9, 1934.)

147. Professor J. F. Ritt: *On the order of a system of algebraic differential equations.*

This paper presents rigorous results relative to the number of arbitrary constants in the solution of a finite system of algebraic differential equations. The orders to which the unknowns appear in the equations being given, upper bounds are secured for the number of constants in the solutions of the irreducible systems into which the given system decomposes. (Received March 10, 1934.)

148. Dr. I. J. Schoenberg: *On asymptotic distributions of arithmetical functions.*

In 1928 the author proved (Mathematische Zeitschrift, vol. 28) that the sequence $\phi(n)/n$ $(n=1, 2, 3, \cdots; \phi(n)$ is the Euler function) has a continuous asymptotic distribution over the interval (0, 1). Using the author's method, H. Davenport proved recently (Sitzungsberichte der Preussischen Akademie, 1933) the following general theorem: Let the arithmetical function $f(n)$ satisfy the following conditions: (1) $0<f(n)\leqq 1$; (2) $f(mn)=f(m)f(n)$ if $(m, n)=1$; (3) with positive constants C, c we have $0\leqq f(p^{e-1})-f(p^e)\leqq Cp^{-ce}$ for all primes p and $e\geqq 1$; (4) the numbers $\log f(p)$ are linearly independent for all sufficiently large primes p. Then the sequence $f(n)$, $(n=1, 2, 3, \cdots)$, admits a continuous asymptotic distribution over (0, 1). In the present note, by essentially different methods, the following theorem is derived. If $f(n)$ satisfies the conditions (1′) $f(n)>0$, (2) as above, (3′) the series $\sum_p |\log f(p)|/p$ converges, then $f(n)$ has already an asymptotic distribution over $(0, +\infty)$ which however need not be continuous. This distribution is certainly continuous if also (4) holds. It may be emphasized that this last theorem makes no assumptions whatever about the values $f(p^e)$ for $e>1$, except that they be positive. (Received March 9, 1934.)

149. Professor Oystein Ore: *Remarks on polynomials taking prime values.*

Pólya, A. Brauer, H. Dorwart, and the author have investigated the irreducibility of polynomials taking a fixed prime value a certain number of times. In the present paper the results are generalized to polynomials taking any prime values. The exact upper limit for the number of prime values which a reducible polynomial may take is obtained. Certain criteria of a different type for irreducibility are also derived. (Received February 28, 1934.)

150. Dr. G. A. Hedlund (National Research Fellow): *A metrically transitive system.*

It has been known for some time that there exist transitive geodesics on closed orientable surfaces of genus greater than one and of everywhere negative curvature. The question as to whether regional transitivity implies metrical transitivity has not been answered. But it can be shown that in the case of a certain set of surfaces which are closed, orientable, of genus greater than one, and of constant negative curvature, the system of geodesics has the property of metrical transitivity. The result is described in detail in a note *On the metrical transitivity of the geodesics on a surface of constant negative curvature*, in the February issue (1934) of the Proceedings of the National Academy of Sciences. (Received February 20, 1934.)

151. Dr. A. B. Brown: *Note on the form of a first-order partial differential equation.*

A simple direct proof is given of the possibility of replacing a first-order partial differential equation by one which does not involve the dependent vari-

able explicitly. The usual proof by use of the "complete integral" uses an elaborate machinery and requires a considerable amount of detailed work to be made complete. (Received February 5, 1934.)

152. Professor O. J. Farrell: *On approximation to an analytic function by polynomials.*

The chief results obtained are as follows: (1) if the function $f(z)$ is analytic and bounded in a finite simply connected region R whose boundary divides the z-plane into just two regions, there exist polynomials $p_n(z)$, $n=1, 2, \cdots$, that converge to $f(z)$ in R as $n\to\infty$ in such a way that the limit (as $n\to\infty$) of the least upper bound of $|p_n(z)|$ for z in R does not exceed the least upper bound of $|f(z)|$ for z in R; (2) if the function $f(z)$ is analytic in a finite simply connected region R whose boundary divides the z-plane into just two regions and if the double integral over R of the pth power $(0<p)$ of $|f(z)|$ exists, then there exist polynomials $p_n(z)$, $n=1, 2, \cdots$, which converge to $f(z)$ in R as $n\to\infty$ in such a way that the limit (as $n\to\infty$) of the double integral over R of the pth power of $|f(z)-p_n(z)|$ is zero. A comparison of the measures of convergence involved in these two theorems is made in the case of a particular example. It is shown that the first of these theorems cannot be extended to the most general finite simply connected region. (Received February 21, 1934.)

153. Dr. E. K. Haviland: *On the determinateness of the multi-dimensional momentum problem.*

Let ϕ_i, $i=1, 2$, be distribution functions, i.e., monotone absolutely additive set functions defined throughout the xy-plane, S, and possessing the total variation 1 there. Furthermore, suppose the momenta of ϕ_1 and ϕ_2 exist and are the same, so that $G_{pq}=\int\int_S x^p y^q d_{xy}\phi_i(E)$ for $i=1, 2$ and all non-negative values of the integers p and q. Then a sufficient condition that $\phi_1=\phi_2$ is that the $2m$th root of $\sum_{r=0}^{2m}\binom{2m}{r}|G_{2m-r,r}|=o(m)$. Moreover, the condition is almost necessary in that there are examples showing that ϕ_1 need not equal ϕ_2 if $o(m)$ is replaced by $o(m^{1+\epsilon})$. This theorem may easily be extended to n-dimensional spaces, $n>2$, and may be applied to a deduction of the multi-dimensional Gaussian distribution. (Received March 9, 1934.)

154. Mr. F. J. Murray: *Partial differential operators as linear transformations between Hilbert spaces.*

We define a function space $\mathfrak{W}$, in which the elements are functions of (x, y) on a bounded region S and the scalar product is $(u, v)=\int\int_S(u_{xx}\bar{v}_{xx}+u_{xy}\bar{v}_{xy}+u_{yx}\bar{v}_{yx}+u_{yy}\bar{v}_{yy}+u_x\bar{v}_x+u_y\bar{v}_y+u\bar{v})dS$. We show that $\mathfrak{W}$ is a Hilbert space. A linear partial differential expression $L(u)$ of the second order with bounded measurable coefficients represents a bounded transformations from $\mathfrak{W}$ to $\mathfrak{L}_2$, the Hilbert space of scalar product $(u, v)=\int\int_S u\bar{v}dS$. We then further limit B, the boundary of S, to be a rectifiable curve with certain other restrictions. Let $\beta(u)$ be a linear combination of the boundary values of u, u_x, and u_y, $u\epsilon\mathfrak{W}$, with coefficients which are bounded measurable functions of the arc-length s. Then $w=\beta(u)$ represents a bounded transformation from $\mathfrak{W}$ to $\mathfrak{G}$, the space

of functions defined on the boundary, of scalar product $(u, v)=\int_S u\bar{v}ds$. We then give methods for the computation of all $u\epsilon\mathfrak{B}$, such that $L(u)=v$ and $\beta(u)=w$ for a given $v\epsilon\mathfrak{L}_2$ and $w\epsilon\mathfrak{G}$. (Received March 7, 1934.)

155. Professor John von Neumann: *Almost periodic functions in a group.*

In this paper the notion of almost periodic functions due to H. Bohr is generalized to arbitrary groups G. By varying the form of the definition of these functions due to S. Bochner, the notion becomes entirely independent of continuity, or of any sort of topology in G. The main difficulty is to define an equivalent of H. Bohr's "integral mean." This is done by an entirely new process of defining the "mean" (which, for instance, in the Bohr case, applies even to non-measurable functions). After this has been done, the operational method of Weyl can be applied, and leads to the "fundamental" and "approximation" theorems, corresponding to those of Bohr. The theory which is thus obtained permits generalization of the Frobenius-Schur theory of representations, beyond its extension by Weyl-Peter, to all bounded representations of any group G. The familiar theorems of completeness hold. The continuity of all expansion terms of continuous almost periodic functions is established (if G is topological), and various special G's are discussed. The existence of a "maximal" set of "characteristic" and of almost periodic functions (in a certain precise sense) is proved for locally compact Abelian groups by the use of Haar's generalized integral, and various theorems of the author on operators. (Received February 23, 1934.)

156. Professor J. F. Ritt: *Algebraic difference equations.*

It was known in the second half of the eighteenth century that non-linear algebraic difference equations, even if algebraically irreducible, might have more than one solution depending on an arbitrary periodic function. This paper gives a bound for the number of such solutions. (Received March 10, 1934.)

157. Professors George Rutledge and R. D. Douglass: *A hypergeometric parametric function defining the range of de la Vallée Poussin summation.*

The functions $Q(x, z)=F(x, -x, 1, z)$ and $-\partial Q/\partial z$ are integral functions of x defined by Stirling interpolation series with z as parameter, $-1\leqq z<1$. For $z=-1$, these functions are denoted by $Q(x)$, $xM_1(x)$, respectively. They have positive integer values for positive integer values of x, and as x becomes infinite, $x=n$, $Q(n)\cong 2^{1/2}M_1(n)\cong 2^{1/4}\cosh[\alpha n/(\pi n)^{1/2}]$, where $\alpha=2\sinh^{-1}1=\log(3+2^{3/2})$. $(VP)\sum_0^\infty u_n$ and $u_0+(VP')\sum_1^\infty u_n$ denote identical processes of summation. (See *The inverse matrix for de la Vallée Poussin summation*, Journal of Mathematics and Physics of the Massachusetts Institute of Technology, vol. 11 (1932), pp. 73–82.) It is convenient to consider $(VP')\sum_1^\infty u_n$. The (VP') transforms of the divergent alternating series $\sum_1^\infty(-1)^{n-1}Q(n)$, $\sum_1^\infty(-1)^{n-1}2M_1(n)/n$ are, respectively, $\sum_1^\infty(-1)^{n-1}$, $\sum_1^\infty(-1)^{n-1}1/n$. The function $Q(n)$ serves the same purpose for (VP') summation (and hence also

for (VP) summation) as the function n^k for Cesàro C_k summation. A necessary condition that $\sum_1^\infty u_n$ shall be (VP') summable is that $u_n = o\{Q(n)\}$. $Q(n) = 2, 8, 38, 192, 1002, \cdots$, and $M_1(n) = 1, 5, 25, 129, 681, \cdots$, satisfy simple recursion formulas. (Received February 19, 1934.)

158. Miss Norma Stelford and Professor H. A. Simmons: *Classes of maximum numbers associated with certain symmetric equations in n reciprocals.*

In a previous paper dealing with this subject (Transactions of this Society, vol. 34, no. 4, p. 876) the authors considered certain symmetric equations with right members of the form b/a, $a \equiv (c+1)b-1$, and thus equal to a sum of two unit fractions. In the present paper a much milder restriction is made on the nature of b/a, and analogs of the theorems relative to maximum numbers in the article referred to above are still obtained. Thus the main results of that article are generalized considerably. (Received February 17, 1934.)

159. Professor H. A. Simmons: *The Kellogg solution of a cyclo-symmetric equation; questions concerning extreme numbers relative to this solution.*

Let $C_t(1/x)$ be the following cyclo-symmetric function of the n reciprocals $(1/x_i)$, $(i=1, 2, \cdots, n)$, $n>t$: $C_t(1/x) = (1/x_1 \cdots x_t) + (1/x_2 \cdots x_{t+1}) + \cdots + (1/x_n x_1 \cdots x_{t-1})$. In this paper the author obtains the Kellogg solution (Transactions of this Society, vol. 34, no. 4, p. 884) of the equation (1), $C_r(1/x) + \lambda_{r+1} C_{r+1}(1/x) + \cdots + \lambda_s C_s(1/x) = b/[(c+1)b-1]$, where the λ's, b, c are integers ≥ 0. Then certain maximum numbers and certain minimum numbers relative to special cases of equation (1) are identified. Certain problems which remain unsolved are proposed for (1) and other more general equations somewhat like (1). (Received February 17, 1934.)

160. Mr. F. C. Smith: *On the asymptotic developments of analytic functions.*

In a series of papers published during the years 1900–1908, E. W. Barnes obtained the asymptotic developments of a large number of function types by means of highly specialized methods. Systematic methods of obtaining such asymptotic expansions have been developed in more recent years by W. B. Ford and C. V. Newsom. In this paper, the general theory is used in considering the asymptotic behavior of the following function types: (1) $f(z) = \sum_{n=0}^\infty z^n/(n+\theta)^\beta$; (2) $f(z) = \sum_{n=0}^\infty h(n) z^n/\Gamma(n+p)$. (Received February 24, 1934.)

161. Dr. J. I. Vass: *A class of boundary problems of highly irregular type.*

J. W. Hopkins (Transactions of this Society, vol. 20 (1919), p. 245) and L. E. Ward (Annals of Mathematics, vol. 26 (1925), p. 21; also, Transactions of this Society, vol. 29 (1927), p. 716) have considered the problem of the expansion of a function $f(x)$ in a series of solutions of ordinary linear differential

systems, in which the differential equation, $d^nu/dx^n+\rho^nu=0$, $n\geqq 3$, (ρ a complex parameter) is associated with boundary conditions of highly irregular type. The present paper takes up the expansion problem for certain highly irregular systems in which the order is reduced, $n=2$, and the element of symmetry, present in the solutions of the above equation, plays no important role. Two distinct systems involving the equation $d^2u/dx^2-2\rho\cos c\, du/dx+\rho^2u=0$, $c=(p/q)\pi$, $0<2p<q$, are discussed with the purpose of exhibiting the true nature of such expansion problems. For the systems considered, sufficient conditions upon $f(x)$ are developed under which the expansion for the function converges uniformly or is summable by an application of the Riesz typical means. Comparisons are made with the work of Hopkins and Ward and also with the corresponding situations for systems which are regular or mildly irregular. (Received February 28, 1934.)

162. Professor C. C. MacDuffee and Mr. E. D. Jenkins: *A substitute for the Euclid algorithm in algebraic fields.*

A method free from tentative steps is outlined for computing the greatest common divisors (provided they exist) of two or more integral numbers of any quadratic field. The method involves the use of matrices with rational integral elements. Except for one step involving the solution of a diophantine equation, the method is applicable to algebraic fields of higher order, and a tentative method will often give at least one greatest common divisor in these cases. To show the ease and rapidity of calculation by this method, examples are given in $F[(53)^{1/2}]$ and in $F(\theta)$ where $\theta^3+6\theta+8=0$, the latter being a cubic field of class number 3. (Received March 1, 1934.)

163. Mr. Rufus Oldenburger: *Canonical triples of bilinear forms.*

In this paper are given the canonical forms to which triples of bilinear forms can be reduced under non-singular linear transformations in the complex field where a pair of bilinear forms in each triple has associated with it distinct characteristic roots. Complete systems of invariants for triples are obtained from the canonical forms. (Received March 2, 1934.)

164. Mr. Rufus Oldenburger: *Factorization of hyperspace matrices into a product of two-way matrices one of which is singular.*

In another paper the author has given necessary and sufficient conditions for a matrix to be of the form $(a^{(1)}_{\alpha i_1}\cdots a^{(n)}_{\alpha i_n})$, where the matrices $(a^{(1)}_{\alpha i_1},\cdots,a^{(n)}_{\alpha i_n})$ are non-singular on α (rows are linearly independent). In the present paper necessary and sufficient conditions are obtained for a matrix to be of the above form where $(a^{(1)}_{\alpha i_1})$ is singular on α, and $(a^{(2)}_{\alpha i_2}),\cdots,(a^{(n)}_{\alpha i_n})$ are non-singular on α. In solving the problem it is necessary to obtain necessary and sufficient conditions for a set of matrices to be simultaneously equivalent under non-singular linear transformations to a set of diagonal matrices. These conditions are given in this paper. (Received March 2, 1934.)

165. Mr. Rufus Oldenburger: *Non-singular multilinear forms.*

A multilinear form equivalent to $x_\alpha y_\alpha \cdots z_\alpha w_\alpha$ is called a *non-singular form*. Let $\delta=(\delta_{ij\cdots kl})$, where $\delta_{ij\cdots kl}=1$ for $i=j=\cdots=k=l=1, \cdots, n$, $\delta_{ij\cdots kl}=0$ for $(i, j, \cdots, k, l)\neq(1, 1, \cdots, 1, 1), \cdots, (n, n, \cdots, n, n)$. The matrix δ and all matrices equivalent to δ under non-singular linear transformations are called *non-singular matrices*. Necessary and sufficient conditions for the non-singularity of a trilinear form have been given in another paper by the author. In the present paper such conditions are given for the non-singularity of an r-linear form, where $r\geqq 4$. The determination of these conditions depends largely on the determination of necessary and sufficient conditions for an m-tuple of multilinear forms, where one of the forms is non-singular, to be equivalent to a set of multilinear forms with diagonal matrices. (Diagonal matrices are matrices whose only non-vanishing elements are elements determined by setting all indices equal.) It is shown that a matrix $(g_{i\cdots l})$ is non-singular if and only if it can be written in the form $(a_{\alpha i}\cdots d_{\alpha l})$, where $(a_{\alpha i}), \cdots, (d_{\alpha l})$ are non-singular two-way matrices. Finally, necessary and sufficient conditions for a matrix to be of the form $(a_{\alpha i}\cdots d_{\alpha l})$, where $(a_{\alpha i}), \cdots, (d_{\alpha l})$ are only assumed to be non-singular on α (rows), are written down. (Received March 2, 1934.)

166. Mr. Rufus Oldenburger: *Solution of a class of simultaneous matrix and integral equations.*

It is proved in this paper that the condition $CE'=C'E$ is necessary and sufficient for the existence of a non-singular matrix solution Y of the equations $CY=C'$, $YE=E'$, where C, C', E, E' are non-vanishing vectors. This property is generalized to denumerably infinite matrices and integral equations, and related to the problem of solving the equations $XAY=A'$, $CY=C'$, $XB=B'$ for non-singular matrices X, Y, where A, A' are non-singular matrices, and C, C', B, B' are vectors. Limiting cases where some of the vectors vanish are also considered. (Received March 2, 1934.)

167. Mr. Rufus Oldenburger: *Space ranks of the multiple composite of hyperspace matrices.*

The composite of two matrices $(a_{\rho i_1\cdots i_m})$, $(b_{\rho j_1\cdots j_n})$ may be formed by combining indices in essentially two ways: by writing $(a_{\rho i_1\cdots i_m}b_{\rho j_1\cdots j_m})$ where ρ is summed, and $(a_{\rho i_1\cdots i_m}b_{\rho j_1\cdots j_n})$ where ρ is not summed. Where the composite of two or more matrices is made by summing on at least one combined index and not summing on at least one other combined index we have the *multiple composite* of these matrices. In this paper certain relations are derived between the space ranks of the multiple composite of a set of matrices, obtained by combining indices in any way, and the space ranks of these matrices. (Received March 2, 1934.)

168. Professor L. E. Dickson: *New universal Waring theorems.*

The content of this paper is sufficiently indicated by the title. (Received March 5, 1934.)

169. Dr. E. F. Beckenbach: *A characteristic property of surfaces of negative curvature.*

It has been pointed out (Transactions of this Society, vol. 35 (1933), p. 663) that if a surface S of non-positive Gaussian curvature K is given in conformal representation, $x_i = x_i(u, v)$ with $\sum x^2_{iu} = \sum x^2_{iv} = \lambda(u, v)$, $\sum x_{iu}x_{iv} = 0$, then $\lambda(u, v)$ is subharmonic (and even that a necessary and sufficient condition that $K \leqq 0$ on S is that $\log \lambda$ be subharmonic). Examples are now given of particular conformal representations of surfaces with $K > 0$ for which $\lambda(u, v)$ is subharmonic; nevertheless, the following converse of the above result is obtained: If for *all* conformal representations of a surface S on (u, v)-domains, $\lambda(u, v)$ is subharmonic, then $K \leqq 0$ on S. As a result, it follows that the familiar inequalities of function theory, $a(r) \geqq \pi r^2 \lambda(u_0, v_0)$, $l(r) \geqq 2\pi r \lambda(u_0, v_0)^{1/2}$, where $a(r)$ and $l(r)$ denote the area and length of boundary of the image of $(u-u_0)^2 + (v-v_0)^2 \leqq r^2$ on S, are characteristic of surfaces with $K \leqq 0$. (Received March 5, 1934.)

170. Dr. R. D. James (National Research Fellow): *The representation of integers as sums of values of cubic polynomials.*

A cubic polynomial in x is an integer for all integers $x \geqq 0$ if and only if it has the form $P(x) = a(x^3-x)/6 + b(x^2-x)/2 + cx + d$, where a, b, c, and d are integers. Under the assumptions that a, b, and c have no common factor, $a > 0$, and $d = 0$, it is proved in this paper that every sufficiently large integer is a sum of nine values of $P(x)$. The method of proof is that of Landau (*Zum Waringschen Problem, Dritte Abhandlung*, Mathematische Zeitschrift, 32 (1930), pp. 699–702). (Received March 5, 1934.)

171. Mr. G. C. Webber: *Waring's problem for cubic functions.*

Recent work by L. E. Dickson has thrown much light on the number of functional values necessary for the representation of an integer as a sum of positive integral values of a cubic function. The author considers certain sets of functions $f(x) = p(x^3-x)/6 + q(x^2-x)/2 + ux$, $p > 0$, $q \neq 0$, p, q, and u being integers. By a substitution $x = X + t$, t an integer, $f(x)$ is transformed into $F(X) = p(X^3-X)/6 + gX + a$. It is proved that every integer exceeding s_0, say, is a sum of nine or ten values of $F(X)$ for values $\geqq |t|$ of X. This enables us to show that every integer exceeding a constant s_1 is a sum of nine or ten values of $f(x)$ for values $\geqq 0$ of x. These constants s_0 and s_1 are readily determined. A separate discussion for cubic functions without square term completes the investigation initiated by Dickson (Transactions of this Society, vol. 36, pp. 1–12). (Received March 5, 1934.)

172. Professor Dunham Jackson: *The summation of series of orthogonal polynomials.*

The summability of a development in series of orthogonal polynomials or orthogonal trigonometric sums by the first arithmetic mean is discussed by a method which involves a somewhat restrictive hypothesis as to the continuity of the function represented, but admits broad generality in the character of the

weight function for which the orthogonal system is defined. (Received March 6, 1934.)

173. Mr. O. K. Sagen: *The integers represented by positive sets of ternary quadratic non-classic forms.*

This paper treats the problem of determining all integers represented by a set of positive ternary quadratic non-classic forms having given determinant. For given a and d, conditions that there exist integers satisfying the equation $d+ar+bs=abc$ are found. From these results it is shown that the set of forms having determinant $2d$ represents all integers except those in certain arithmetical progressions. (Received March 6, 1934.)

174. Professor A. R. Crathorne: *A reduction formula for the moments of a binomial distribution about the origin.*

If m_i is the ith moment of a binomial distribution, $(p+q)^n$, about the origin, then $m_{n+1}=m_1m_n+pq\,dm_n/dp$. The formula is derived by algebraic manipulation from the well known general relation between semi-invariants and moments, together with the more special known relationships among moments and semi-invariants for the binomial distribution. (Received March 8, 1934.)

175. Professor Wilhelm Maier: *Concerning cubic theta functions.*

Following Riemann and Appell, the infinite series $\sum_{h=-\infty}^{+\infty} e^{2\pi i(zh+yh^2/2+xh^3/3)} = \theta\begin{Bmatrix} x \\ y \end{Bmatrix}$, which converges for $0<F(y)$ and $0=F(z)$, defines an analytic function of x, y, and z. To get the analytic continuation of that function for non-real values of z, a new expansion of $\theta\begin{Bmatrix} x \\ y \end{Bmatrix}$ is given in terms of cyclindrical functions. (Received March 8, 1934.)

176. Dr. E. S. Akeley: *On a type of action integral in physics and a certain transformation.*

Let L be an invariant function of a positive definite metric tensor g_{ij} in four independent variables and its associated contracted Riemannian Christoffel tensor R_{ij}. Consider the invariant integral $I=\int Lg^{1/2}dx^{(1)}\cdots dx^{(4)}$ and the associated Euler equations for the calculus of variations problem $\delta I=o(\delta g_{ij})$. We obtain six independent fourth order equations for the ten g_{ij}. Furthermore, consider the transformation from (g_{ij}, R_{ij}) to (λ_m, p_{im}) where $g_{ij}=\sum_m p_{im}p_{jm}$ and $R_{ij}=\sum_m \lambda_m p_{im}p_{jm}$; L becomes a symmetric function of the λ_m and we obtain sixteen second order equations for the twenty $(\lambda_m p_m)$. If, furthermore, the coordinate system is canonicalized (See abstract 140–1–19) so that $\lambda_m=x_{(m)}^{(m)}$ we obtain sixteen independent second order equations for the sixteen functions p_{im}, out of which four first order equations are deducible. If one considers the coordinate system canonicalized in the original integral, the new variation problem is of the form $\delta I=o(\delta p_{im})$ under an auxiliary condition which introduces a Lagrangian tensor. The equations for this problem are shown to be equivalent to the original equations. These results can be easily generalized for a non-definite form in n dimensions. (Received March 8, 1934.)

177. Professor R. E. Langer: *The solutions of the Mathieu equation with a complex variable and at least one parameter large.*

The Mathieu differential equation $u''+(\Delta-\Omega\cos 2z)u=0$ is considered over the complex z plane and the asymptotic forms of its solutions are determined for all cases in which the parameters are real and at least one of them is large. When $|\Omega|\geq|\Delta|$ these forms are subject to the Stokes' phenomenon and this is quantitatively determined. The associated subdivision of the z plane and the forms which describe the solutions in the respective regions depend upon the relative magnitudes of Δ and Ω. The asymptotic form of the characteristic equation for the parameter values which admit of a periodic solution is obtained, and the form of the characteristic exponent in its dependence upon the parameters is given in each case. (Received March 8, 1934.)

178. Mr. Nathan Schwid: *The asymptotic forms of the Hermite and Weber functions.*

The Hermite equation, $d^2U/dZ^2-2Z\,dU/dZ+2KU=0$, and its counterpart under a simple change of variable, the Weber equation, $d^2W/dZ^2+(2K+1-Z^2)W=0$, have been widely discussed. Considerations of the forms of their solutions when the latter are asymptotically dependent upon the parameter K have heretofore been restricted to the case in which K is real, with the variable Z, when complex, limited to a finite strip of the Z plane. This paper considers these equations when the parameter is complex and the variable Z is permitted to vary over the entire complex plane. The asymptotic forms of the solutions with respect to large values of K are derived. The results are obtained by utilizing formulas developed by Langer (Transactions of this Society, vol. 34, No. 3, pp. 447–480) for solutions of an ordinary second order differential equation of a certain general structure, a type to which the Weber equation is readily reduced. (Received March 9, 1934.)

179. Professor H. S. Wall: *Groups of transformations of continued fractions.*

Let $\xi=x_0+[x_n/1]_1^\infty$, $\xi_\alpha=x_0^\alpha+[x_n^\alpha/1]_1^\infty$ be continued fractions with convergents $k_0k_1k_2\cdots$ and $k_\alpha kj\cdots k_\gamma k_{\alpha+p}k_{\beta+p}\cdots k_{\gamma+p}\cdots$, respectively, where $\alpha\beta\cdots\gamma$ is a permutation of $01\cdots p-1$. Let a be the substitution which takes $01\cdots p-1$ into $\alpha\beta\cdots\gamma$, respectively, and let b be another substitution. The numbers x_n^α are certain rational functions of some of the x_n, say $x^\alpha=f^\alpha(x)$. Then $f^\alpha(x^b)=f^\alpha(f^b(x))=f^{ab}(x)=x^{ab}$. For every finite substitution group a simply isomorphic group of birational transformations can be obtained in this way. If ξ converges in an x-region, ξ_α will converge in the corresponding x^α-region. Example: If $p=3$, $a=(012)$, $c=(01)$ then we have j' (identity), $f^a: x^a=-(1+x+y)/x$, $y^a=yz/[(1+x)(1+z)+y]$, $z^a=-(1+x+y)/[(1+x)\cdot(1+z)+y]$; $f: x=-x/(1+x+y)$, $y=-y/(1+x+y)$, $z^c=1/z$; f^a, f^{ac}, f^{ca}. Here $x=x_{2n}$, $y=x_{2n+1}$, $z=x_{2n+2}$. The aggregate of all these transformations ($p=2, 3, 4, \cdots$) is an infinite group. (Received March 9, 1934.)

180. Mr. R. E. Huston: *Generalized asymptotic Waring theorems.*

For every integral exponent $k \geqq 3$ it is proposed to determine what number s of variables and what sets of positive integral coefficients $a_1, \cdots, a_s$ insure a solution of the diophantine equation $n = \sum a_i h_i^k$, $i = 1, \cdots, s$, $h_i \geqq 0$, for every integer $n > b$, where b depends only on k, s, and the maximum of the a_i. For $a_1 = \cdots = a_s = 1$, the problem reduces to the classical Waring problem in asymptotic form. Here the explicit value $s \geqq (k-2)2^{k-1}+5$ was established by Hardy and Littlewood. The Hardy-Littlewood mode of attack can be made to yield this same value of s for our more general problem if certain congruential conditions on the a_i are satisfied. That these conditions are satisfied by very general sets of coefficients is proved by a sufficiency theorem depending only on the primitivity of the coefficients. An example shows, moreover, that for certain exponents at least no more powerful sufficiency theorem in terms of primitivity of the coefficients alone can be obtained. Two other types of sufficiency theorems are developed and the paper concludes with illustrations of the results obtained for $k = 3$, 4, and 5. (Received March 9, 1934.)

181. Mr. B. C. Getchell: *Integration of interval functions.*

The norm and sigma limits of functions of subdivisions as developed by Moore and Smith (American Journal of Mathematics, vol. 44 (1922), pp. 101–121) are applied to functions with linear intervals as arguments. A necessary and sufficient condition is given for the existence of the norm integral when the sigma integral exists. The class of integrable functions is extended by means of sequences to include generalizations of those classes of point functions which are integrable in the Lebesgue or Stieltjes sense. An integral of the latter type is based upon the property of absolute continuity. (Received March 9, 1934.)

182. Mr. J. W. T. Suckau: *On uniform convergence.* Preliminary report.

If a sequence of functions converges on a set S, then the sequence is uniformly convergent on certain subsets of S; e.g., any finite subset. A construction is given which yields all subsets of S on which the convergence is uniform. The object of the paper is twofold. First to investigate the amount of uniform convergence automatically present. It is found that if S is non-denumerable, then there exists a denumerably infinite subset of S on which the convergence is uniform; moreover, an example is given of a sequence of functions which converges on the whole continuum and has the property that on every non-denumerable set the convergence is not uniform. The second object is to restrict the functions and the set S so that uniform convergence is induced on "large" subsets of S. It is found that the restricted functions form a very large class, in fact the class of measurable functions. Applications are made to the Lebesgue theory. (Received March 9, 1934.)

183. Professor H. R. Brahana: *Metabelian groups of order p^{n+m} with commutator subgroups of order p^m.*

The metabelian groups considered in this paper are generated by an abelian group of order p^n and type 1, 1, $\cdots$ and m permutable operators of order p from its group of isomorphisms. The number of distinct abstract groups of this kind with centrals of order p^{n-k} is shown to be equal to the number of conjugate sets of matrices $x_1M_1+x_2M_2+\cdots+x_kM_k$, where the x's are variables and the M's are m-rowed square matrices with elements in a modular field, mod p, under two types of transformation: (a) linear homogeneous transformations on the x's with coefficients in the modular field, and (b) elementary transformations with coefficients in the modular field on the M's simultaneously. Properties of the groups corresponding to invariant factors of $x_1M_1+x_2M_2$, and corresponding to irreducible factors of its determinant are exhibited for $k=2$. These relations obviously generalize to any value of k not greater than $n/2$. (Received March 9, 1934.)

184. Dr. H. P. Thielman: *Note on the use of fractional integration for obtaining expansions in squares of Bessel functions.*

In this note it is shown that fractional integration of Bessel functions will lead in certain cases to the squares of such functions. Use is then made of this fact to derive new expansions in squares of Bessel functions by fractionally integrating known expansions in first powers of such functions. The derived expansions are valid for the interval in which the known expansions converge uniformly. (Received March 9, 1934.)

185. Dr. H. P. Thielman: *An analogue in Hilbert space of a theorem of Arzela for function space.*

Arzela's theorem is as follows: A necessary and sufficient condition on an infinite set of functions that it contain a uniformly convergent subsequence of continuous functions is that the given set contain an infinite subset which is equibounded and equicontinuous. As an analogue of this we obtain the following result: A necessary and sufficient condition on an infinite set of points in Hilbert space to contain a subsequence which converges uniformly relative to a scale function, which is a point in Hilbert space, is that the infinite set contain an infinite subset which is equibounded and equiconvergent. A set $\{x_i^{(\alpha)}\}$ of points in Hilbert space is defined to be equiconvergent if for every $\epsilon>0$ there exists a positive integer N such that for every choice of $\alpha_1^{(i)}$, $\alpha_2^{(i)}$ from the α's we have $\sum_{i=N}^{\infty}[x_i^{(\alpha_1^{(i)})}-x_i^{(\alpha_2^{(i)})}]^2<\epsilon$. (Received March 9, 1934.)

186. Mr. H. S. Kaltenborn: *An extension of Riesz's theorem on linear functional operations.*

The most general form of a linear continuous functional operation on a function $f(x)$ continuous on (a, b) has been expressed by F. Riesz (Annales scientifiques de l'École Normale Supérieure, vol. 31 (1914), pp. 9–14) as a Stieltjes

integral of $f(x)$ with respect to a function of bounded variation. An analogous result is derived in this paper for the case where $f(x)$ has at most discontinuities of the first kind on (a, b). The operation $F[f]$ is expressed in terms of the modified Stieltjes integral considered by B. Dushnik (Thesis, University of Michigan, 1931). If the *norm* or *metric* for this class of functions is defined as the least upper bound of $|f(x)|$ on (a, b), then any linear continuous functional operation on $f(x)$ is expressible in the form $T[f]=\int_a^b f\,d\psi+\sum_{i=1}^{m}[f(x_i)-f(x_i-0)]\phi(x_i)$, where $x_1, x_2, x_3, \cdots$, denote the points of discontinuity of $f(x)$, $\psi(x)$ and $\phi(x)$ depend only on T, and are such that $\psi(x)$ is of bounded variation, and $\phi(x)$ vanishes except at a denumerable set of points and the sum of the absolute values of $\phi(x)$ at these points is finite, and when the integral is of the above mentioned modified Stieltjes type. (Received March 9, 1934.)

187. Professor G. Y. Rainich and Mr. H. E. Vaughan, Jr.: *Variation of product integrals.*

The question considered is that of change of the value of a line product integral corresponding to a change, not affecting the endpoints, of the path. In index notation the matrices appear as A_μ^i and the product integral as $\int A_\mu^i dx^\rho+\delta_j^i$. Corresponding to a given path with the endpoints a and b this will be written as $P_{j(a)}^{i(b)}$, omitting the path in the notation. The formula proved is $\Delta P_{j(a)}^{i(b)}=\int_a^b P_{\rho(x)}^{i(b)} R_{\sigma\lambda\mu}^{\rho} dx^\lambda \Delta x^\mu P_{j(a)}^{\sigma(x)}$ where $R_{n_1pq}^{m}=(\delta/\delta x^q)A_{n_2p}^{m}-(\delta/\delta x^p)A_{n_2q}^{m}+A_{\alpha q}^{m}A_{n_2p}^{\alpha}=A_{\alpha p}^{m}A_{n_2q}^{\alpha}$. The proof is made simpler by the use of a special coordinate system. The application to curved spaces where R is the Riemann tensor is obvious. (Received March 9, 1934.)

188. Professor T. H. Hildebrandt: *On bounded linear functional operations.*

In this paper, the most general linear bounded functional operation on the vector space of all bounded sequences, convergence being uniform convergence, is determined by the use of an integration process based on the general limits of E. H. Moore. The same procedure yields results in a number of other spaces which are not separable. (Received March 9, 1934.)

189. Professor C. N. Moore: *On the relationship between the Abel-Poisson method and that of Cesàro for the summation of multiple series.*

In a recent note in the Bollettino della Unione Matematica Italiana (October, 1933) J. C. Vignaux has considered certain properties of the Abel-Poisson method of summation applied to double series, and he has raised the question as to the relationship between this method and that of Cesàro for such series. In a review of Vignaux's paper in the Zentralblatt (January 27, 1934) E. Kobetliantz has remarked that it would be interesting to have this question settled. It is shown in the present paper that for a multiple series of order n summability $(C, \rho_1, \rho_2, \cdots, \rho_n;\ \rho_i \geqq 0)$ implies summability to the same value

by the Abel-Poisson method. This is a consequence of certain general theorems on convergence factors in multiple series given in a paper by C. N. Moore (Transactions of this Society, vol. 29 (1927), pp. 227–238). (Received March 9, 1934.)

190. Dr. Gordon Pall: *On the order invariants of integral quadratic forms.*

Let A denote double the matrix of an integral quadratic form in s variables, of non-zero determinant, the cross-product coefficients not necessarily even; let $s-2I$ be the signature. For any $k(1\leqq k\leqq s)$, the g.c.d. of all the principal and doubles of the secondary minor determinants of order k in A is an invariant of the class; this g.c.d. is even if k is odd; denote it by $2d_k$ if k is odd, d_k if k is even; define $d_0=1$; define o_k by $d_k^2o_k=4^\epsilon d_{k-1}d_{k+1}(\epsilon=1+(-1)^k)(k=1,\cdots,s-1)$, $o_0=o_s=0$. The o_k are integers satisfying (a) $o_k\not\equiv 2 \pmod 4$; (b) if any $o_k(1\leqq k\leqq s-1)$ is odd, $o_{k-1}\equiv o_{k+1}\equiv 0 \pmod{16}$; (c) if $o_1, o_2, \cdots, o_{s-1}$ are odd (s even), $(-)^{(s-2I)/2}\cdot o_1o_2\cdots o_{s-1}\equiv 1 \pmod 4$; (d) if all the o_k are squares, and $o_i\equiv o_{i+1}\equiv 0 \pmod{16}$ for not more than two values i, then $s-2I\not\equiv \pm 3,4 \pmod 8$. These conditions are necessary and sufficient for a form to exist with the invariants $I, o_1\cdots, o_{s-1}$. Further the kth primitive concomitant of f is properly or improperly primitive according as o_k is even or odd. H. J. S. Smith's conjecture concerning the invariants of the concomitants is proved. (Received February 9, 1934.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

ARNEBERG (J.). Kortfattet laerebok i projeksjonstegning med opgaver for realgymnasiet. Trondheim, F. Bruns Bokhandels Forlag, 1933.

AUMANN (G.). Beiträge zur Theorie der Zerlegungsraüme. (Dissertation.) München, 1932.

BAKER (H. F.). Principles of geometry. Volume 5: Analytical principles of the theory of curves. Cambridge, University Press, 1933. 10+247 pp.

BAVINK (B.). Ergebnisse und Probleme der Naturwissenschaften. Eine Einführung in die heutige Naturphilosophie. 5te Auflage. Leipzig, Hirzel, 1938. 9+650 pp.

BLACK (M.). The nature of mathematics. A critical survey. (International Library of Psychology, Philosophy and Scientific Method.) London, Kegan Paul, and New York, Harcourt, Brace and Company, 1934. 14+219 pp.

BOERNER (H.). Über einige Eigenwertprobleme und ihre Anwendung in der Variationsrechnung. (Dissertation.) Leipzig, 1932.

BRESLICH (E. R.). The administration of mathematics in secondary schools. Chicago, University of Chicago Press, and London, Cambridge University Press, 1933. 8+408 pp.

BROUWER (L. E. J.). See UNIVERSITY OF AMSTERDAM.

BRUCE (W. H.). The nine circles of the triangle. Denton, North Texas State Teachers College, 1932. 48 pp.

BRUNSTÄD (F.). Logik. München und Berlin, Oldenbourg, 1933. 99 pp.

BRUSS (K.). Über Flächen mit einer Schar von Haupttangentenkurven, die aus Schraubenlinien auf parallelen Zylindern besteht. (Dissertation.) Jena, 1932. 81 pp.

BUSEMANN (H.). Über die Geometrien, in denen die "Kreise mit unendlichem Radius" die kürzesten Linien sind. (Dissertation.) Göttingen, 1932.

CARONNET (T.). Exercices de géométrie (compléments). Paris, Vuibert, 1933. 180 pp.

CAULLERY (M.). La science française depuis le XVIIe siècle. Paris, Armand Colin, 1933. 214 pp.

CLAY (J.). See UNIVERSITY OF AMSTERDAM.

COUFFIGNAL (L.). Les machines à calculer. Leurs principes, leur évolution. Paris, Gauthier-Villars, 1933. 9+86 pp.

CRAMER (W.). Die Reziprozitätsformel für Gaussche Summen in reell quadratischen Zahlkörpern. (Dissertation.) Breslau, 1932. 27 pp.

DEURING (M.). Theorie der algebraischen Funktionen. (Dissertation.) Göttingen, 1932.

DULAC (H.). Points singuliers des équations différentiels. (Mémorial des Sciences Mathématiques, No. 61.) Paris, Gauthier-Villars, 1933.

DURELL (C. V.) and SIDDONS (A. W.). Graph book. An exercise book and text book. Revised edition. London, Bell, 1933. 66+14 pp.

ESTÈVE (R.) et MITAULT (H.). Compléments d'algèbre. Paris, Gauthier-Villars, 1933. 75 pp.

EUCLID. Die Elemente. Nach Heibergs Text aus dem Griechischen übersetzt und herausgegeben von C. Thaer. Teil I: Buch I–III; Teil II: Buch IV–VI. (Ostwalds Klassiker, Band 235–236.) Leipzig, Akademische Verlagsgesellschaft, 1933. 88+75 pp.

FORD (L. R.). Differential equations. New York and London, McGraw-Hill, 1933. 10+263 pp.

GRUNSKY (H.). Neue Abschätzungen zur konformen Abbildung ein- und mehrfach zusammenhängender Bereiche. (Dissertation.) Berlin, 1932.

DE HARTOG (A. H.). See UNIVERSITY OF AMSTERDAM.

HJELMSLEV (J.). Geometrisk Analyse. II: Elementaere Funktioner. III: Vektoranalyse. København, Gjellerup, 1933. 116 pp.+141 pp.

HURWITZ (A.). Mathematische Werke. Band 2. Basel, Birkhäuser Verlag, 1933.

KERST (B.). Mathematische Spiele. Berlin, G. Grote, 1933. 6+90 pp.

KNOPP (K.). See VON MANGOLDT (H.).

KRAUS (W.). Über den Zusammenhang einiger Charakteristiken eines einfach zusammenhängenden Bereiches mit der Kreisabbildung. (Dissertation.) Giessen, 1932. 28 pp.

LUDOLPH (G. L.) en POTMA (A. P.). Coördinatenleer voor het middelbaar technisch onderwijs en voor zelfstudie. Groningen, J. B. Wolters, 1932. 212 pp.

VON MANGOLDT (H.) und KNOPP (K.). Einführung in die höhere Mathematik. 6te Auflage. Band III. Leipzig, Hirzel, 1933. 16+618 pp.

MANNOURY (G.). See UNIVERSITY OF AMSTERDAM.

MILNE-THOMPSON (L. M.). The calculus of finite differences. London, Macmillan, 1933. 23+558 pp.

MITAULT (H.). See ESTÈVE (R.).

MORLEY (F.) and MORLEY (F. V.). Inversive geometry. Boston, Ginn, and London, Bell, 1933. 9+273 pp.

MORLEY (F. V.). See MORLEY (F.).

MORRIS (C. R.). Idealistic logic. A study of its aim, method and achievement. London, Macmillan, 1933. 9+338 pp.

MOUFANG (R.). Zur Struktur der projektiven Geometrie der Ebene. (Dissertation.) Frankfurt, 1932.

MÜNZNER (H.). Günstigste Bestimmung der Umkehrung der Laplace-Transformierten zur Auffindung verborgener Periodizitäten. (Dissertation.) Göttingen, 1932. 41 pp.

NAHM (E.). Über den Vergleich von Komplexen geometrischer Gebilde und tonfreier Farben. (Dissertation.) Frankfurt, 1932. 49 pp.

VAN DER NEUT (D. N.). Limitaties en sommaties. (Dissertation, Leiden Amsterdam, H. J. Paris, 1933. 88 pp.

PERCIVAL (A. S.). Mathematical facts and formulae. London, Blackie, 1933. 6+125 pp.

POS (H. J.). See UNIVERSITY OF AMSTERDAM.

POTMA (A. P.). See LUDOLPH (G. L.).

RAFF (H.). Beschränkte divergente Folgen und reguläre Matrizen. (Dissertation.) Tübingen, 1932. 34 pp.

RAY (B. S.). Berechnung der Eigenwerte des asymmetrischen Kreisels. (Dissertation.) Göttingen, 1932.

REESTMAN (B. M.). Inleiding tot de theorie der klassenlichamen. (Dissertation, Leiden.) Amsterdam, H. J. Paris, 1933. 104 pp.

RÉVÉSZ (G.). See UNIVERSITY OF AMSTERDAM.

REY (A.). La jeunesse de la science grecque. Paris, La Renaissance du Livre, 1933. 13+537 pp.

RINOW (W.). Über Zusammenhänge zwischen der Differentialgeometrie im grossen und im kleinen. (Dissertation.) Berlin, 1932.

ROHRBACH (H.). Die Charaktere der binären Kongruenzgruppen moderner p^2. (Dissertation.) Berlin, 1932.

RÜCKERT (W.). Zum Eliminationsproblem der Potenzreihenideale. (Dissertation.) Freiberg i. Br., 1932. 23 pp.

SALJÉ (H.). Zur Abschätzung der Fourierkoeffizienten ganzer Modulformen. (Dissertation.) Leipzig, 1932.

SALKOWSKI (E.). Der Gruppenbegriff als Ordnungsprinzip des geometrischen Unterrichts. 2te, erweiterte Auflage. Leipzig, Teubner, 1933. 68 pp.

SCHOLTEN (P.). See UNIVERSITY OF AMSTERDAM.

SCHÜRMANN (P.). Über monotone Funktionen. (Dissertation.) Münster, 1932. 18 pp.

SER (J.). Les calculs formels des séries de factorielles. Paris, Gauthier-Villars, 1933. 8+99 pp.

SEVERI (F.). Lezioni di analisi. Volume 1. Bologna, Zanichelli, 1933. 8+434 pp.

SIDDIQI (M. R.). Zur Theorie der nichtlinearen partiellen Differentialgleichungen vom parabolischen Typus. (Dissertation.) Leipzig, 1932.

SIDDONS (A. W.). See DURELL (C. V.).

SPECHT (W.). Eine Verallgemeinerung des symmetrischen Gruppe. (Dissertation.) Berlin, 1932. 32 pp.

SPERNER (E.). Über die fixpunktfreien Abbildungen der Ebene. (Hamburger Mathematische Einzelschriften, Heft 14.) Leipzig, Teubner, 1933. 47 pp.

THAER (C.). See EUCLID.

TUBERGEN (J.). See UNIVERSITY OF AMSTERDAM.

UNIVERSITY OF AMSTERDAM. De uitdrukkingswijze der wetenschap. Kennistheoretische openbare voordrachten gehouden aan de Universiteit te Amsterdam 1932–1933. Van G. Mannoury, H. J. Pos, J. D. van der Waals, L. E. J. Brouwer, P. Scholten, J. Tubergen, J. Clay, R. R. Welschen, G. Révész, A. H. de Hartog. Groningen, Noordhoff, 1933. 117 pp.

VAN DER WAALS (J. D.). See UNIVERSITY OF AMSTERDAM.

WARSCHAWSKI (S.). Über das Randverhalten der Ableitung der Abbildungsfunktion bei konformer Abbildung. (Dissertation.) Basel, 1932.

WELSCHEN (R. R.). See UNIVERSITY OF AMSTERDAM.

WILLIS (E. J.). Spherical analytic geometry. Richmond, Va., William Byrd Press, and Glasgow, Brown Son and Ferguson, 1933. 45 pp.

PART II. APPLIED MATHEMATICS

ABRAHAM (M.). Theorie der Electrizität. Neubearbeitet von R. Becker. 6te vollständig neubearbeitete Auflage. Bank 2: Elektronentheorie. Leipzig, Teubner, 1933. 7+400 pp.

ANDERSON (J. S.). See KING (A.).

ANDREI (I.). L'erreur du concept d'"électron." Identité de l'électron avec l'atome d'hydrogène. Preuves expérimentales. Bucarest, Tipografia "Bokovina" I. E. Toroutiu, 1933.

ASTON (F. W.). Mass-spectra and isotopes. London, Arnold, 1933. 12+248 pp.

AUERBACH (F.). Das naturwissenschaftliche Weltbild. Berlin-Lichterfelde, Hugo Bermüller, 1933. 135 pp.

AUGER (P.), BAUER (E.), DE BROGLIE (L.), et COURTINES (M.). Les bases expérimentales immédiates de la théorie des quanta. Paris, Hermann, 1933. 29 pp.

BARKHAUSEN (H.). Les tubes à vide et leurs applications. Tome 1: Principes généraux. Traduit et adapté de l'allemand par A. Lourie. Paris, Dunod, 1933. 172 pp.

BAUER (E.). Introduction à la théorie des groupes et à ses applications à la physique quantique. Paris, Les Presses Universitaires de France, 1933. 170 pp.

——— See AUGER (P.).

BECKER (R.). See ABRAHAM (M.).

BIED-CHARRETON (R.). De la turbine à l'atome. Préface de C. Fabry. Paris, Gauthier-Villars, 1933. 6+199 pp.

BLOCH (F.). See MARX (E.).

BLOCH (L.). Précis d'électricité théorique. 2e édition, revue et corrigée. Paris, Gauthier-Villars, 1933. 7+476 pp.

BRAGG (W.) and BRAGG (W. L.). The crystalline state. Volume 1: A general survey, by W. L. Bragg. London, Bell, 1933. 14+352 pp.

BRAGG (W. L.). See BRAGG (W.).

DE BROGLIE (L.). See AUGER (P.).

BRUHAT (G.). Cours de mécanique physique à l'usage de l'enseignement supérieure scientifique et technique. Paris, Masson, 1934. 708 pp.

CASIMIR (H.). See EHRENFEST (P.).

CHERRY (F. H.). Descriptive geometry. An introduction to engineering graphics. (Engineering Science Series.) New York, Macmillan, 1933. 11+127 pp.

COMRIE (L. J.). The Hollerith and Powers tabulating machines. London, the author, 1933. 48 pp.

COPEL (P.). Eléments d'optique géométrique à l'usage des candidats aux grandes écoles. Paris, Gauthier-Villars, 1933. 8+206 pp.

COURTINES (M.). See AUGER (P.).

CURCHOD (A.). Memento d'électrotechnique. Tome III: Réseaux de distribution d'énergie électrique. Transmission et distribution de l'énergie électrique. Production de l'énergie électrique d'origine thermique et d'origine hydraulique. Législation. Paris, Dunod, 1933. 643 pp.

DARROW (K. K.). Elementare Einführung in die Quantenmechanik. Aus dem Englischen übersetzt von E. Rabinowitsch. Leipzig, Hirzel, 1933. 6+123 pp.

DAURE (P.). Introduction à l'étude de l'effet Raman. Ses applications chimiques. Paris, Editions de la Revue d'Optique Théorique et Instrumentale, 1933. 8+90 pp.

DEANS (W. M.). See GRIMSEHL (E.).

DEBYE (P.). Struktur der Materie. Vier Vorträge. Leipzig, Hirzel, 1933. 4+50 pp.

——— See MARX (E.).

DRUCKER (C.). See WIEN (W.).

DRURY (F. E.) and HASLAM (W. T.). Mathematics for students of building. London, Arnold, 1933. 271 pp.

EHRENFEST (P.). Golfmechanika. Bewerkt door Dr. H. Casimir. 's-Gravenhage, W. P. van Stockum en Zoon, 1932. 103 pp.

EUCKEN (A.) und WOLF (K. L.), herausgegeben von. Hand- und Jahrbuch der chemischen Physik. Band 1: Theorien des Aufbaues der Materie. Abschnitt 1: Die Grundlagen der Quantentheorie, von H. A. Kramers. Leipzig, Akademische Verlagsgesellschaft, 1933. 222 pp.

——— Hand- und Jahrbuch der chemischen Physik. Band 6: Elektrizität und Materie. Abschnitt 2: Elektrische Leitfähigkeit. Abschnitt 3: Positive Korpuskularstrahlen. Leipzig, Akademische Verlagsgesellschaft, 1933. 344+284 pp.

EVE (A. S.) and KEYS (D. A.). Applied geophysics in the search for minerals. 2d edition. Cambridge, University Press, 1933. 11+296 pp.

FABRY (C.). See BIED-CHARRETON (R.).

FAJANS (K.). See WIEN (W.).

FARADAY (M.). Faraday's diary. Printed and published for the first time, under the editorial supervision of Thomas Martin. Volume 3: May 26, 1836–Nov. 9, 1839. Volume 4: Nov. 12, 1839–June 26, 1847. London, Bell, 1933. 12+466+12+448 pp.

FROST (E. B.). An astronomer's life. Boston and New York, Houghton Mifflin, 1933. 11+300 pp.

GEIGER (H.) und SCHEEL (K.), herausgegeben von. Handbuch der Physik. 2te Auflage. Band 23, Teil 2: Röntgenstrahlung ausschliesslich Röntgenoptik. Redigiert von H. Geiger. Berlin, Springer, 1933. 9+541 pp.

GEYER (E. W.). See SCHÜLE (W.).

GLASSTONE (S.). Recent advances in physical chemistry. 2d edition. London, J. and A. Churchill, 1933. 8+498 pp.

GRIMSEHL (E.). Lehrbuch der Physik zum Gebrauch beim Unterricht, neben akademischen Vorlesungen und zum Selbststudium. Band 1. Leipzig, Teubner, 1933. 700 pp.

——— A textbook of physics. Edited by R. Tomaschek. Authorized translation from the 7th German edition by W. M. Deans. Volume 4: Optics. London, Glasgow, and Bombay, Blackie, 1933. 12+301 pp.

HAMILTON (W. R.). Abhandlungen zur Strahlenoptik. Übersetzt und mit Anmerkungen herausgegeben von G. Prange. Leipzig, Akademische Verlagsgesellschaft, 1933. 572 pp.

HARMS (F.). See WIEN (W.).

HASLAM (W. T.). See DRURY (F. E.).

HEITLER (W.). See MARX (E.).

HERZENBERG (A.). See ROTHE (R.).

HOBART (H. M.). See PUNGA (F.).

HUTCHINSON (R. W.). Wireless. Its principles and practice. 2d edition. London, University Tutorial Press, 1933. 12+308 pp.

JAEGER (C.). Théorie générale du coup de bélier. Application au calcul des conduites à caractéristiques multiples et des chambres d'équilibre. Paris, Dunod, 1933. 268 pp.

JOHN (H.). Kinematische Modelle der irdischen Trägheitsbewegung. (Dissertation.) Karlsruhe, 1932. 42 pp.

KEHREN (E.). Die Anwendung der konformen Abbildung in der Elektrostatik. (Dissertation.) Aachen, 1932.

KESSELRING (F.). The elements of switchgear design. Translated from the German by S. R. Mellonie and J. Solomon. London, Pitman, 1932. 7+182 pp.

KEYS (D. A.). See EVE (A. S.).

KING (A.) and ANDERSON (J. S.). Chemical calculations. Their theory and practice. London, Murby, and New York, Van Nostrand, 1933. 10+181 pp.

KOENIG (F. O.). See WIEN (W.).

KRAMERS (H. A.). See EUCKEN (A.).

KREMANN (R.). See WIEN (W.).

KRONING (R. DE L.). See MARX (E.).

LANGE (E.). See WIEN (W.).

LEHMANN (W.). Die Elektrotechnik und die elektromotorischen Antriebe. Berlin, Springer, 1933. 302 pp.

LINDOW (M.). Differentialrechnung unter Berücksichtigung der praktische Anwendung in der Technik. 5te Auflage. (Mathematisch-Physikalische Bibliothek.) Leipzig, Teubner, 1933. 102 pp.

LINDSAY (R. B.). Physical mechanics. An intermediate text for students of the physical sciences. London, Chapman and Hall, 1933. 10+436 pp.

LOURIE (A.). See BARKHAUSEN (H.).

MALLET (E.). Vectors for electrical engineers. London, Chapman and Hall, 1933. 8+181 pp.

MARTIN (D. J.). An introduction to thermodynamics. For chemists. London, Arnold, 1933. 7+343 pp.

MARTIN (T.). See FARADAY (M.).

MARX (E.), herausgegeben von. Handbuch der Radiologie. 2te Auflage. Band 6: Quantenmechanik der Materie und Strahlung. Teil 2: Moleküle. Bearbeitet von R. de L. Kroning, P. Debye und H. Sack, F. Bloch, G. Placzek und W. Heitler. Leipzig, Akademische Verlagsgesellschaft, 1933. 612 pp.

MATTEI (A.). Delle resistenze di rinculo in un sistema di freno-ricuperatore. Roma, Revista di Artiglieria e Genio, 1932.

MELLONIE (S. R.). See KESSELRING (F.).

MITTELSTÄDT (G. S.). Die kosmischen Strahlen. Plauen, Kommissionsverlag A. Kells, 1933.

MOORFIELD (S. H.) and WINSTANLEY (H. H.). Mechanics and applied heat. For engineers. London, Arnold, 1933. 330 pp.

NIELSEN (J.). Forelaesninger over rationel Mekanik. I: Statik og Kinematik. København, Gjellerup, 1933. 222 pp.

OLLENDORFF (F.). See ROTHE (R.).

PHILLIPS (J. E.). Intermediate magnetism and electricity. London, Christophers, 1933. 11+365 pp.

PILLEY (J.). Electricity. Oxford, Clarendon Press, and London, Oxford University Press, 1933. 14+348 pp.

OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, March 30-31, 1934.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than March 9. Abstracts received by the Secretaries by that date will appear in the March issue of the BULLETIN. By invitation of the program committee, there will be a symposium on *Algebraic geometry*, under the leadership of Professors Solomon Lefschetz, Oscar Zariski, A. A. Albert, Virgil Snyder, and A. B. Coble.

CHICAGO, ILLINOIS, April 6-7, 1934.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than March 9. Abstracts received by the Secretaries by that date will appear in the March issue of the BULLETIN. At this meeting, Professor R. E. Langer will give an address on *The asymptotic solutions of linear ordinary differential equations.*

BERKELEY, CALIFORNIA, June 20, 1934.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, Calif., not later than May 30. Abstracts received by the Secretaries by May 6 will appear in the May issue of the BULLETIN.

WILLIAMSTOWN, MASSACHUSETTS, SUMMER MEETING AND COLLOQUIUM, September 4-7, 1934.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than August 1. Abstracts received by July 6 will appear in the July issue of the Bulletin. Professor Norbert Wiener will deliver a series of colloquium lectures which were planned by himself and the late Dr. R. E. A. C. Paley on *Topics from harmonic analysis in the complex domain.*

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1932, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

WHOLE NUMBER 414

CONTENTS

PAGE

The Annual Meeting in Cambridge. By R. G. D. Richardson 177

International Mathematical Congress Medals. By J. L. Synge 189

Veblen on Projective Relativity. By D. J. Struik 191

Radó on Plateau's Problem. By Jesse Douglas 194

The California Colloquium. By J. M. Thomas 197

Shorter Notices 201

Notes 204

Abstracts of Papers 209

New Publications 235

For official Communications and Notices, see the inside of the back cover.

GEORGE BANTA PUBLISHING COMPANY, MENASHA, WISCONSIN